Realization of the Chua Family of New Nonlinear Network Elements Using the Current Conveyor

Introduction

In a recent paper, Chua [1] introduced three new network elements, namely, the mutator, the scalor, and the reflector. These, together with the rotator that he introduced earlier [2], form a family of four basic elements that prove to be useful in the area of nonlinear network synthesis as well as suggesting many unusual circuit applications. This correspondence extends the work of Chua by providing a more unified treatment of the circuit realization of the four elements. The current conveyor [3] will be used to obtain implementations that are in closer match to the ideal equations defining the elements.

A General Realization Technique for the Mutator and the Scalor

We will first show that four of the six mutators together with both scalors can be realized by a linear four-port and two passive elements. The passive elements are either two resistors or a resistor and a capacitor, with inductors being avoided. Consider the linear four-port shown symbolically in Fig. 1(a) and characterized by

$$\begin{bmatrix} v_1 \\ i_2 \\ v_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \\ i_3 \\ v_4 \end{bmatrix} \tag{1}$$

where all v and i are total instantaneous values of voltages and currents.

Using the characterization of the second-generation current conveyor [3] (CC II),[1] it can be easily shown that the linear four-port defined in (1) can be realized by two complementary CC II's as shown in Fig. 1(b). For this reason the linear four-port is termed the composite current conveyor (CCC). Note that the symbol for CCC suggested in Fig. 1(a) reflects the fact that ports 1 and 3 set the currents at ports 2 and 4, respectively, with the ⊖ and ⊕ symbols indicating the direction of this setting. These facts follow from (1).

There are a finite number of ways in which this four-port can be converted to a two-port by connecting either a resistance or a capacitance across each of the two other ports. Three combinations of passive elements are of interest, namely, R–R, R–C, and C–R. By doing an exhaustive tabulation of all the possible two-ports realized in this manner, some are found to be redundant and should be directly eliminated. Although some other cases appear also to be equivalent, there exist fundamental differences regarding their stability behavior. This correspondence does not consider the stability problems of the elements realized, however, it is felt that they all possess what is equivalent to open-circuit stability (OCS) or short-circuit stability (SCS) existing, for example, in all practical realizations of negative-impedance converters. Therefore, for each element there always exist two dual realizations.

Table I lists the mutators and scalors realized from CCC with the four-port connections shown and Chua's equivalent realizations indicated. Some other cases not listed in Table I result in realizations of some of the listed elements with a shift of the characteristics from the first quadrant to the third. These may be of interest in some applications.

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[1] Patent was applied for.

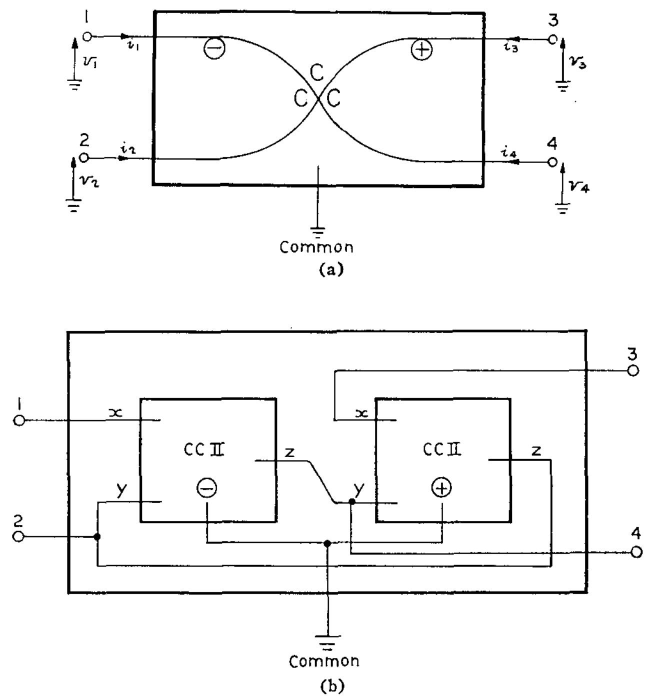


Fig. 1. (a) Symbolic representation of the composite current conveyor. (b) An implementation of CCC using two complementary current conveyors.

The two remaining mutators, namely, the type-2 L–R mutator and the type-2 L–C mutator, cannot be realized from CCC if inductors are to be avoided. Realizations of these are given in Table II each using three current conveyors.

Realization of the Rotator and the Reflector

It has been shown before [2] that a rotator can generally be realized by a negative-impedance converter (NIC) and three positive passive elements. The NIC used should be direct-coupled and moreover should retain its characteristics over a large signal range passing through the origin. Since either version of the current conveyor [3] can provide such an NIC, a rotator can be realized by a single current conveyor and three passive elements.

By utilizing the unique three-port NIC [3] constructed from the current conveyor, accurate reflector realizations using two current conveyors and some passive elements are possible. Fig. 2 shows two such realizations for an R reflector with an indication of the corresponding equivalent circuit given in [1].

Conclusions

We may conclude with the following remarks.

1) Accurate straightforward realization of Chua's new network elements are possible in terms of the current conveyor. As well as providing a unified approach to design, the current conveyor can be readily integrated, and as such is commercially attractive. Moreover, the conveyor has fundamentally a much wider frequency band than available operational amplifiers, as was demonstrated by some of its applications [4].

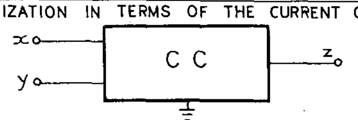
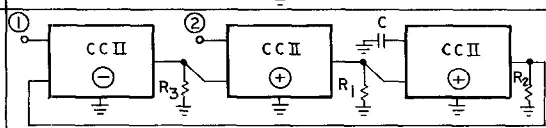
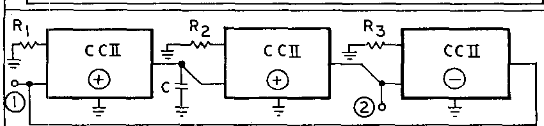
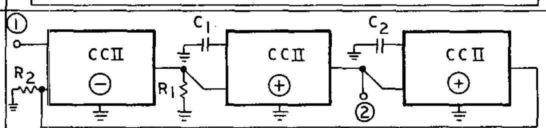
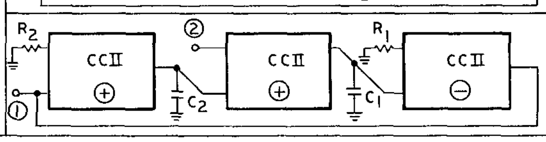
2) Chua [1] suggests the use of the L–R mutator to provide a variable inductance tuned by a resistance variation. However, this application is not confined to the L–R mutator as any active realization of the gyrator, when loaded by a capacitance, can provide a variable inductance by varying the gyrator con-

TABLE I
USE OF CCC TO REALIZE MUTATORS AND SCALORS

Element Realized	Port Connections of (CCC)* 1	2	3	4	Corresponding Chua's Realization†	Transfer Equations
L–R Mutator Type-1	(1)	R	C	(2)*	(3)†	$v_1 = CR(dv_2/dt)$
	(2)	C	R	(1)	(4)	$i_1 = -i_2$
C–R Mutator Type-1	(1)	(2)	R	C	(3)	$v_1 = v_2$
	(2)	(1)	C	R	(4)	$i_1 = -CR(di_2/dt)$
C–R Mutator Type-2	(1)	R	(2)	C	(1)	$v_1 = -i_2R$
	C	(2)	R	(1)	(2)	$i_1 = C(dv_2/dt)$
L–C Mutator Type-1	(1)	R_1	(2)	R_2	(1)	$v_1 = -i_2R_1$
	R_2	(2)	R_1	(1)	(2)	$i_1 = (v_2/R_2)$
Voltage Scalor	(1)	K_vR	R	(2)	(3)	$v_1 = K_vv_2$
	(2)	R	K_vR	(1)	(4)	$i_1 = -i_2$
Current Scalor	(2)	(1)	R	K_IR	(3)	$v_1 = v_2$
	(1)	(2)	K_IR	R	(4)	$i_1 = -K_Ii_2$

* Numbers (1) and (2) correspond to the two ports of the element as defined by Chua [1].
† Numbers (1), (2), (3), and (4) correspond to the different equivalent circuit realizations as defined by Chua [1].

TABLE II
REALIZATION OF THE TYPE-2 L–R MUTATOR AND THE TYPE-2 L–C MUTATOR

ELEMENT REALIZED	REALIZATION IN TERMS OF THE CURRENT CONVEYOR 	CORRESPOND'G CHUA REALIZATION	TRANSFER EQUATION
TYPE 2 L–R MUTATOR		(1)^2	$V_1 = -C R_1 R_2 \frac{dI_2}{dt}$
		(2)	$I_1 = \frac{V_2}{R_3}$
TYPE 2 L–C MUTATOR		(1)	$V_1 = C_2 R_2 \frac{dV_2}{dt}$
		(2)	$I_1 = -\frac{1}{C_1R_1}\int I_2 dt$

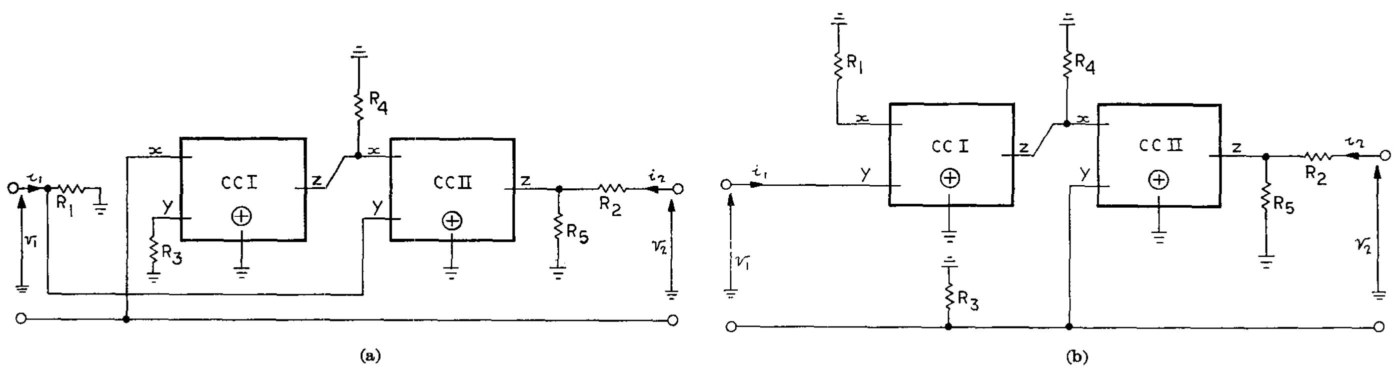


Fig. 2. Realization for the R reflector corresponding to the T-equivalent circuits given by Chua. (a) $0 < \theta < 90°$; $R_1 = R \cot \theta$; $R_2 = R \tan \theta$; $R_3 = R \text{ CSC } 2\theta$, $K = 2R \text{ CSC } 2\theta = -R_5(1 - R_1/R_4)$. (b) $90° < \theta < 180°$; $R_1 = -R \cot \theta$; $R_2 = -R \cot \theta$; $R_3 = -R \text{ CSC } 2\theta$, $K = 2R \text{ CSC } 2\theta = R_5(1 - R_3/R_4)$.

stant that is usually of the form of the product of two resistances. It might also be noted that the gyrator is a type-1 L–C mutator.

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[3] A. Sedra and K. C. Smith, "A second-generation current conveyor and its applications," this issue, pp. 132–134.
[4] K. C. Smith and A. Sedra, "A new simple wide-band current-measuring device," *IEEE Trans. Instrumentation and Measurement*, vol. IM-18, pp. 125–128, June 1969.

Continuous Equivalence of General (*RLGC*) Nonuniform Transmission Lines

INTRODUCTION

In the past, equivalence conditions for RC lines and also LC lines have been derived by several authors [1], [2]. In this correspondence, necessary and sufficient conditions for two general ($RLGC$) nonuniform transmission lines (NUTL) to be equivalent continuously along the line are derived from a simple physical requirement. The result is easily extended to the multilayer lines. It is also proved that equivalence of two NUTL's as a two-port network requires continuous equivalence. Finally, it is pointed out that electrically different characteristics cannot be obtained from a certain class of generalized lines since they turn out to be continuously equivalent lines.

CONDITIONS FOR CONTINUOUS EQUIVALENCE

Consider a general NUTL in which $z(x) = r(x) + sl(x)$ and $y(x) = g(x) + sc(x)$ are, respectively, the series impedance and shunt admittance per unit length at the distance x. Also let $Z(u) = R(u) + sL(u)$, $Y(u) = G(u) + sC(u)$ and u be the corresponding quantities for a second line. Then if the two lines are to be "equivalent continuously" along the line, starting at the origin (the choice of the starting point is, of course, arbitrary), first of all, to an incremental section of the first line at $x = 0$, there must exist a corresponding incremental section of the second line at $u = 0$, such that the total series impedance and shunt admittance in the two sections are the same. This is true for all successive small sections of the two lines. In the limit, as the incremental sections become infinitesimally small, this obvious physical requirement can be expressed as

$$[r(x) + sl(x)]\, dx = [R(u) + sL(u)]\, du \tag{1a}$$

and

$$[g(x) + sc(x)]\, dx = [G(u) + sC(u)]\, du. \tag{1b}$$

If these conditions are to hold for all frequencies,

$$\frac{du}{dx} = \frac{r(x)}{R(u)} = \frac{l(x)}{L(u)} = \frac{g(x)}{G(u)} = \frac{c(x)}{C(u)}. \tag{2}$$

The coordinate relation between u and x may also be expressed by

$$\frac{du}{dx} = \sqrt{\frac{l(x)c(x)}{L(u)C(u)}} \tag{3a}$$

or

$$\int_0^u \sqrt{L(u)C(u)}\, du = \int_0^x \sqrt{l(x)c(x)}\, dx \tag{3b}$$

or

$$\int_0^u L(u)\, du = \int_0^x l(x)\, dx. \tag{3c}$$

In (3a) and (3b) any two of the four kinds of line parameters may be used, and in (3c) any one kind of line parameter may also be used. It is apparent that the conditions expressed in (1) or (2) are sufficient for the continuous equivalence stated above.

GENERATION OF EQUIVALENT LINES

In the analysis, synthesis, and practical construction of NUTL's, it is often useful to find some equivalent line from a given line. Suppose we have a NUTL that is characterized by the line parameters $r(x)$, $l(x)$, $g(x)$, and $c(x)$ and the line length d. In generating continuously equivalent lines, we have to satisfy four conditions, namely the four equalities of (2). On the other hand, we have five quantities to be determined, namely, the coordinate u(as a function of x) in addition to the four line parameters $R(u)$, $L(u)$, $G(u)$, and $C(u)$. (As will be seen later, the length of the second line is determined once the coordinate relation $u(x)$ is known.) This allows us one degree of freedom in generating equivalent lines, and hence an infinite number of continuously equivalent lines are possible. This one degree of freedom can be utilized to generate equivalent lines by several different methods. For example, 1) we can preassign the coordinate relation $u = u(x)$ or $x = x(u)$ in an arbitrary way, as long as the monotonicity condition is met [see (3)]. Then, expressing x as a function of u and substituting into the following relations (4), we can determine the variations of all the four parameters of the equivalent line.

$$\begin{aligned} R(u) &= r(x)\cdot\frac{dx}{du} \\ L(u) &= l(x)\cdot\frac{dx}{du} \\ G(u) &= g(x)\cdot\frac{dx}{du} \\ C(u) &= c(x)\cdot\frac{dx}{du}. \end{aligned} \tag{4}$$

The new line length D is determined by finding the point $u = D$ corresponding to the point $x = d$ using (5).

$$D = u(x)\,\big|_{x=d} \tag{5}$$

or from the requirement that the total quantity of any one kind of parameter in the two lines must be the same [see (3c)], e.g., from (6),

$$\int_0^D C(u)\, du = \int_0^d c(x)\, dx. \tag{6}$$

2) We can preassign the variation of one of the line parameters $R(u)$, $L(u)$, $G(u)$, and $C(u)$ in an arbitrary way. For the simplest case in which one line parameter is constant, for example, when $L(u)$ = constant = L_0, the coordinate relation is determined from (3c) as

$$u = \frac{1}{L_0}\int_0^x l(x)\, dx \tag{7}$$

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