A Feedback Model for Active *n*-Port Networks Derived from a Concept of Port Orientation

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Abstract—Beginning with fundamental observations, the concept of port orientation is introduced providing a classification of networks as oriented or nonoriented. By incorporating the orientation property in the network mathematical description, a feedback model equivalent to active networks is derived. In this way the stability criteria of feedback systems can be directly applied to active networks.

The feedback model is applied to the stability study of negative resistance 1-ports, negative impedance converters, and negative impedance inverters. It is shown that the observed stability behavior can be systematically derived without explicitly including the device parasitics. Accordingly, the identity of the negative impedance converter ports as open-circuit stable or closed-circuit stable becomes systematically available.

I. Introduction

In the study of the stability behavior of active networks there appears to be no single unifying approach. Accordingly, the stability properties of some devices such as the negative impedance converter are not completely explained [1]–[3]. On the other hand, in the feedback control discipline it is clear that the study of stability is based on more solid ground. A possible conclusion is that there may be a missing network property whose omission sometimes hinders the successful application of the wealth of stability concepts in feedback theory to active networks. In an attempt to provide such a link, this paper, beginning with fundamental observations, introduces the concept of port orientation. According to this concept an n-port network can be classified as an oriented or a nonoriented network. All practical active networks are either oriented or they possess a part that is oriented. Roughly speaking, the concept of orientation as presented can be related to the unidirectional signal flow in feedback systems. In this sense it can be regarded as a recognition of the presence of the cause–effect property in practical active networks. It can also be related to the unilateral property of some elements as discussed by Bode [4, pp. 533–561].

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By incorporating the property of orientation in the mathematical characterization of the network we shall proceed to derive a feedback model that precisely represents any oriented network. In this way the stability criteria of feedback systems can be directly applied to the feedback model of active n-ports.

As an application to the proposed formulation, the stability of negative resistance circuits will be considered in some detail. This will allow the systematic derivation of the stability properties of elements such as negative resistance 1-ports, negative impedance converters (NIC), and negative impedance inverters (NIV).

II. Port Orientation and Oriented *n*-Ports

In modern network theory an n-port network is completely defined by the totality of admissible signal pairs (voltage and current) that the network can support [4, pp. 1–8], [5]. To investigate some of the network properties such as causality and stability, one has to specify the excitation mode of the network, namely, the way in which the $2n$-port signals are constrained by the network termination. For convenience, if a port is externally connected to an ideal voltage source it will be labeled voltage-excitation current response (VECR), while if a port is externally connected to an ideal current source it will be labeled current-excitation voltage response (CEVR).

In the most general case a port may be connected to a source having a finite source impedance. Indeed except in rare pathological cases, any n independent functions of the $2n$-port signals can be taken as excitation. Fortunately, however, it will be shown that for the study of the network behavior under any termination, or correspondingly under any general mode of excitation, it is sufficient to know the network behavior only for the idealized modes of excitation. Since in the ideal case a port can be either CEVR or VECR, there are $N=2^n$ ideal excitation modes for an n-port network.

We now proceed to define an oriented n-port network.

Definition: An oriented n-port network is an n-port for which only one of the possible N ideal excitation modes results in a single-valued response. Otherwise the network will be called nonoriented.

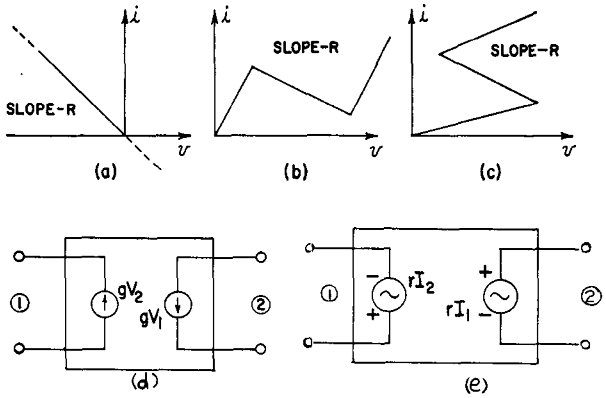


Fig. 1. Examples of oriented and nonoriented networks.

The unique ideal excitation mode of an oriented network will be denoted by $\mathcal{E}$. An example of such an excitation mode $\mathcal{E}$ would be.

$$\mathcal{E} = (v_1, v_2, \cdots, v_k, i_{k+1}, i_{k+2}, \cdots, i_n)$$

which means that the only ideal excitation mode under which the particular n-port network has single-valued response is VECR at ports $1, 2, \cdots, k$, and CEVR at ports $k+1, \cdots, n$. If a port m is represented by v_m in the mode expression, it will be called a voltage-oriented port. On the other hand, if it is represented by i_m it will be called a current-oriented port.

To find whether a particular network is nonoriented it is sufficient to prove that there exists more than one mode of the N idealized excitation modes under which the network provides a single-valued response. On the other hand, to prove that a network is oriented, one ideally needs to exhaustively test all the N idealized modes and prove that one and only one mode provides a single-valued response. However, in practical situations this is a relatively simple task, as will be seen from the examples.

Examples

1) A negative resistance 1-port device having the ideal i–v characteristic shown in Fig. 1(a) can accept both types of idealized excitation, namely, VECR and CEVR, resulting in a single-valued response in each case. Therefore such a 1-port network is a nonoriented one. However, a negative resistance 1-port having the type N i–v characteristic shown in Fig. 1(b) will have a 3-valued response under CEVR. Thus such a port is a voltage-oriented one, and hence, the 1-port network is an oriented network with $\mathcal{E}=(v_1)$. By a dual argument, the negative resistance device having the type S i–v characteristic of Fig. 1(c) is seen to be an oriented network with $\mathcal{E}=(i_1)$.

2) A resistive T (with all positive resistors) is obviously a nonoriented network since it will have a single-valued predictable response under any of the four possible ideal excitation modes.

3) An ideal passive transformer is a nonoriented network since it will have a single-valued predictable response under two (out of the possible four) ideal excitation modes, namely (v_1, i_2) and (i_1, v_2).

4) An ideal voltage amplifier ($R_i=\infty$ and $R_0=0$) is an oriented network with $\mathcal{E}=(v_1, i_2)$.

5) An ideal gyrator for which the circuit in Fig. 1(d) is a small signal model is an oriented network for which $\mathcal{E}=(v_1, v_2)$. To show that, consider, for example, the excitation (i_1, i_2). It might be argued that mathematically the solution $(v_1, v_2) =(g/i_2, -g/i_1)$ exists. However, this situation is physically unattainable because of the impossibility of connecting two current sources in series. A similar argument applies to the excitations (v_1, i_2) and (i_1, v_2). In a dual way the ideal gyrator of Fig. 1(e) is an oriented network with $\mathcal{E}=(i_1, i_2)$.

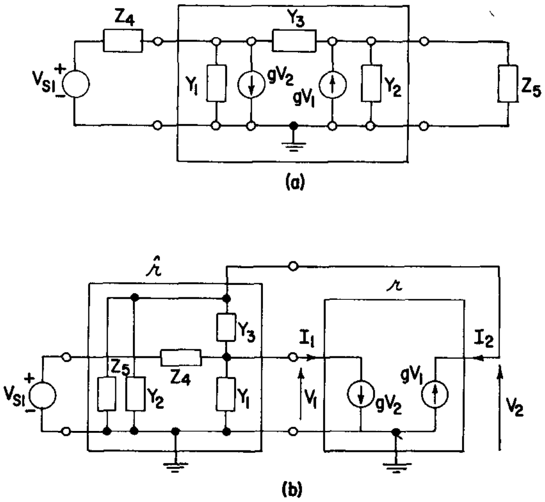


Fig. 2. (a) Nonoriented network (real gyrator). (b) Oriented part $\mathfrak{N}$ (ideal gyrator) and coupling network $\hat{\mathfrak{N}}$ of (a).

An active network which is nonoriented can, in all practical cases, be subdivided such that one of its constituent parts is oriented. This property allows one to find a feedback model equivalent of the network, as demonstrated in Section IV. As an example consider the nonideal gyrator shown in Fig. 2(a). This 2-port as it stands is a nonoriented network. However, an ideal part of the network, namely the perfect gyrator, can be identified. Thus the ideal gyrator forms an oriented network $\mathfrak{N}$ while the rest of the real gyrator can be considered as a coupling 3-port network $\hat{\mathfrak{N}}$ [Fig. 2(b)]. In this way we obtain an oriented network for which a feedback model can be derived.

III. Network Characterization by Unidirectional Equations

Having determined the excitation mode $\mathcal{E}$ for an oriented network, we seek a way to include such information in the matrix description of the network. Consider an oriented linear time-invariant n-port network having the excitation mode

$$\mathcal{E} = (v_1, v_2, \cdots, v_m, i_{m+1}, i_{m+2}, \cdots, i_n). \tag{1}$$

In the frequency domain such a network is described by

$$N(p) = H(p)M(p) \tag{2}$$

where

$$M(p) = \begin{bmatrix} V_m(p) \\ I_{n-m}(p) \end{bmatrix} \tag{3}$$

$$N(p) = \begin{bmatrix} I_m(p) \\ V_{n-m}(p) \end{bmatrix}. \tag{4}$$

$H(p)$ is the $n \times n$ hybrid matrix description of $\mathfrak{N}$ and p is the complex frequency variable. To include the orientation property in the network description, we propose to replace the equality sign in (2) by the unidirectional equality $:=$ across which arithmetic manipulation is not allowed. Thus the network $\mathfrak{N}$ will be described by

$$N(p): = H(p)M(p) \tag{5}$$

where $M(p)$ and $N(p)$ are still given by (3) and (4).

As an example of the application of the unidirectional equality the gyrator of Fig. 1(d) will be described by

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} : = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

while the gyrator of Fig. 1(e) will be described by

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} : = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}.$$

In this manner the difference between the two realizations of the ideal gyrator is implemented in their mathematical representation.

IV. A Feedback Model Equivalent to an Oriented n-Port Network

First, consider the multivariable feedback system represented symbolically in Fig. 3(a) where $R(p)$, $E(p)$, and $C(p)$ are n-dimensional vectors and $G(p)$ and $B(p)$ are $n \times n$ matrices. The diagram clearly defines each of these quantities. Following our notation, the system of Fig. 3(a) can be precisely represented by

$$C(p): = G(p)E(p) \tag{6}$$

$$E(p): = -B(p)C(p) + R(p). \tag{7}$$

It is apparent that the unidirectional equality serves to incorporate the direction of the signal flow in the system's mathematical model.

Next, consider the oriented n-port network $\mathfrak{N}$ described by (5). Let the network $\mathfrak{N}$ be terminated as shown in Fig. 3(b) where the choice between termination in a voltage source with a series impedance or a current source with a parallel admittance is made only to simplify subsequent manipulations. The terminating impedances and admittances should satisfy the following condition:

All the impedances should be nonoriented or current-oriented while all the admittances should be nonoriented or voltage-oriented. In other words, an oriented port should not be terminated in a port having identical orientation. This implies that all the terminating impedances and admittances in the preceding representation should have finite values. Furthermore, some cases such as the series connection of two tunnel diodes across an ideal voltage source would be eliminated.

Assuming that this condition is satisfied, the termination of the network can be described by

$$\begin{bmatrix} V_m(p) \\ I_{n-m}(p) \end{bmatrix} : = -\begin{bmatrix} Z(p) & 0 \\ 0 & Y(p) \end{bmatrix} \begin{bmatrix} I_m(p) \\ V_{n-m}(p) \end{bmatrix} + \begin{bmatrix} V_{sm}(p) \\ I_{s(n-m)}(p) \end{bmatrix} \tag{8}$$

where

$$V_{sm}(p) = [V_{s1}(p)V_{s2}(p) \cdots V_{sm}(p)]^T$$

$$I_{s(n-m)}(p) = [I_{s(m+1)}(p)I_{s(m+2)}(p) \cdots I_{sn}(p)]^T$$

$$Z(p) = \begin{bmatrix} Z_1(p) & 0 & 0 \\ 0 & Z_2(p) & 0 \\ 0 & 0 & Z_m(p) \end{bmatrix} \tag{9}$$

$$Y(p) = \begin{bmatrix} Y_{m+1}(p) & 0 & 0 \\ 0 & Y_{m+2}(p) & 0 \\ 0 & 0 & Y_n(p) \end{bmatrix}. \tag{10}$$

Note that the unidirectional equality in (8) is used to include the case where some of the Z or Y are oriented 1-ports. If this is not the case, the unidirectional equality can be replaced by an equality sign, in which case the following discussion would be unaltered.

Substituting (3) and (4) in (8) we obtain

$$M(p): = -\begin{bmatrix} Z(p) & 0 \\ 0 & Y(p) \end{bmatrix} N(p) + \begin{bmatrix} V_{sm}(p) \\ I_{s(n-m)}(p) \end{bmatrix}. \tag{11}$$

The different terms in (5) and (11) representing the network $\hat{\mathfrak{N}}$ and its termination can be directly identified with the corresponding terms in (6) and (7) representing the multivariable feedback system of Fig. 3(a). This correspondence can be stated as follows:

An oriented linear time-invariant n-port network described by the unidirectional equation (5) with $M(p)$ and $N(p)$ as given in (3) and (4), when terminated as described by (8) [Fig. 3(b)] with $Z(p)$ and $Y(p)$ as in (9) and (10), can be regarded as an n-variable feedback system similar to that in Fig. 3(a) with

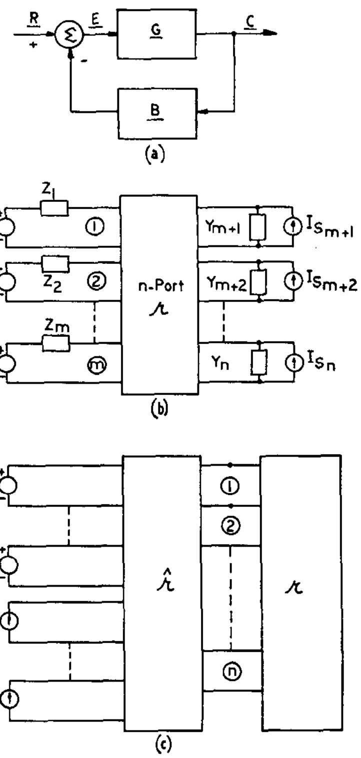


Fig. 3. (a) Multivariable feedback system. (b) Terminated oriented n-port. (c) Oriented n-port $\mathfrak{N}$ coupled to ideal sources through a coupling network $\hat{\mathfrak{N}}$.

correspondences as follows:

$$R(p) = \begin{bmatrix} V_{sm}(p) \\ I_{s(n-m)}(p) \end{bmatrix} \tag{12}$$

$$E(p) = M(p) \tag{13}$$

$$C(p) = N(p) \tag{14}$$

$$G(p) = H(p) \tag{15}$$

$$B(p) = \begin{bmatrix} Z(p) & 0 \\ 0 & Y(p) \end{bmatrix}. \tag{16}$$

The preceding derivation applies for the case in which the network is an oriented one. As mentioned before, many active networks are not oriented at their proper ports but can be broken into an oriented part and a coupling part. Therefore we seek a feedback model to represent the general situation shown in Fig. 3(c). In this case $\mathfrak{N}$ is an oriented n-port, described by (5), coupled to r voltage sources and q current sources through an $(n+r+q)$-port coupling network $\hat{\mathfrak{N}}$. The coupling network can be any linear time-invariant network satisfying the following condition.

Ports 1 to m and $(n+r+1)$ to $(n+r+q)$ should be current oriented or nonoriented, while ports $(m+1)$ to $(m+r)$ should be voltage oriented or nonoriented.

This condition corresponds to the condition imposed on the Z and Y in the case of the simple terminations described before.

In a similar fashion it can be shown that $\mathfrak{N}$ is equivalent to an n-variable feedback system as before, but with the correspondences (12) and (16) replaced by (17) and (18), respectively,

$$R(p) = H^{12}_{n\times(r+q)}(p) \begin{bmatrix} V_{sr}(p) \\ I_{sq}(p) \end{bmatrix} \tag{17}$$

$$B(p) = -H^{11}_{n\times n}(p) \tag{18}$$

where H^{11} and H^{12} are two components of the partitioned hybrid matrix describing the $(n+r+q)$-port coupling network $\hat{\mathfrak{N}}$. This description can be written as

$$\begin{bmatrix} V_m(p) \\ I_{n-m}(p) \\ I_r(p) \\ V_q(p) \end{bmatrix} := \begin{bmatrix} H^{11}_{n\times n}(p) & H^{12}_{n\times(r+q)}(p) \\ H^{21}_{(r+q)\times n}(p) & H^{22}_{(r+q)\times(r+q)}(p) \end{bmatrix} \cdot \begin{bmatrix} I_m(p) \\ V_{n-m}(p) \\ V_{sr}(p) \\ I_{sq}(p) \end{bmatrix} \tag{19}$$

where $I_1, I_2, \cdots, I_n$ are assumed positive in the outward direction of $\hat{\mathfrak{N}}$.

Examples

1) A type N i–v device [Fig. 1(b)] operating on a linearized segment of the negative resistance portion can be described by

$$I := -(1/R)V.$$

If the device is fed with a voltage source V_s in series with a resistance R_s, it will be equivalent to a single-variable feedback system having the parameters

$$G = -1/R$$
$$B = R_s. \tag{20}$$

On the other hand, a type S negative resistance device [Fig. 1(c)] operating on a linearized segment (slope $-R$) of its negative resistance characteristic can be described by

$$V := -RI.$$

If such a device is terminated in a voltage source V_s in series with a resistance R_s, the feedback model parameters will be

$$G = -R$$
$$B = 1/R_s. \tag{21}$$

2) The nonideal gyrator of Fig. 2(a) is shown in Fig. 2(b) separated into an ideal gyrator $\mathfrak{N}$ coupled to the source $V_{s1}(p)$ through a 3-port coupling network $\hat{\mathfrak{N}}$. The feedback model

will be a 2-variable one having

$$G(p) = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$$

$$B(p) = \begin{bmatrix} -1\Big/\left[Y_1 + Y_4 + \dfrac{1}{Z_3 + 1/(Y_2 + Y_5)}\right] & -1\Big/\left\{(Y_1 + Y_4)\left[1 + (Y_2 + Y_5)\left(Z_3 + \dfrac{1}{Y_1 + Y_4}\right)\right]\right\} \\ -1\Big/\left\{(Y_2 + Y_5)\left[1 + (Y_1 + Y_4)\left(Z_3 + \dfrac{1}{Y_2 + Y_5}\right)\right]\right\} & -1\Big/\left[Y_2 + Y_4 + \dfrac{1}{Z_3 + 1/(Y_1 + Y_4)}\right] \end{bmatrix}.$$

V. Application to the Stability Study of Negative Resistance Circuits

In this section the feedback model is applied to the stability study of negative resistance networks enabling a systematic derivation of the stability criteria in each case.

First, consider briefly the stability of the multivariable feedback system shown in Fig. 3(a). The characteristic equation of such a system is

$$\det [1 + G(p)B(p)] = 0. \tag{22}$$

For the system to be stable, the roots of (22) should all lie in the left half of the complex frequency p-plane. Any of the known methods such as the Routh or Nyquist criteria or the root locus technique can be used to study these roots.

In some cases when dealing with idealized system models the dynamics do not explicitly appear in the system description. Correspondingly, G and B become real matrices, and a difficulty arises in applying the preceding condition. Simple reasoning, however, alleviates such a difficulty. Consider a single-variable feedback loop with a real loop gain GB. This system is stable if and only if [11], [12]

$$1 + GB > 0.$$

Extrapolation of this well-known condition to the multivariable case suggests the stability condition

$$\det (1 + GB) > 0. \tag{23}$$

Although there does not seem to be a rigorous mathematical proof for (23), its application always leads to experimentally verifiable results.

Note that this condition does not preclude the role of the system dynamics in determining its stability. In fact, they are accounted for by the implicit assumption of the existence of a finite delay around the loop.

A. 1-Port Negative Resistance Devices

The study of the stability of 1-port negative resistance devices is usually approached by the addition of infinitesimal energy storage elements within the device [6], [7]. One can tailor the solution by including the correct type of parasitic in order to yield results in agreement with practice. However, it is clear that there is no solid argument as to why the particular parasitic element is chosen. Also, since the results thus obtained do not depend upon how small these added elements are, it would appear that there should be a more systematic method to explain the stability behavior of these devices without reverting to such a weakly justified technique.

Existing physical negative resistance 1-port devices exhibit i–v characteristics belonging to one of the two forms shown in Fig. 1(b) and (c). Each of the two types of characteristics represents an oriented network with the type N device being voltage oriented and the type S being current oriented. A type N device operating on a linearized segment (of slope $-R$) of its negative resistance curve, and fed with voltage source V_s in series with a resistance R_s, is equivalent to a single-variable feedback system having the parameters (20). Application of the stability criteria above yields

$$R_s/R < 1, \qquad \text{i.e., } R_s < R$$

for stable operation, which agrees with practice [7].

In a similar manner for a type S device represented by the feedback model (21), stable operation is obtained for

$$R/R_s < 1, \qquad \text{i.e., } R_s > R$$

which again agrees with practice.

It is interesting to note that although no parasitics have been explicitly included in the preceding analysis, their effect on the stability is accounted for by the device orientation as well as the implicit assumption of the existence of delay in a resistive feedback loop. However, explicit inclusion of parasitics in the form of a more elaborate circuit model would still be necessary should it be required to obtain more information about the circuit in oscillation, e.g., the frequency of oscillation. In this case the feedback model can still be applied using (22).

B. Negative Impedance Converters

It has been experimentally found that the NIC exhibits asymmetrical stability behavior at its two ports. That is, one port is open-circuit unstable (OCUS) while the other is short-circuit unstable (SCUS) [1], [8]. Recently it has been shown that direct coupled NICs will always have this asymmetrical stability behavior [1]. Still the proof is unnecessarily complicated by the explicit assumption of the existence of parasitics. However, it has been claimed that an ac coupled NIC with symmetrical stability behavior is possible [2].

As in the case of the negative resistance 1-port, it is possible to arrive at the measured stability characteristics of NICs in a systematic manner without explicit reference to parasitics. A physical realization of the ideal current-inversion NIC can be characterized by either one of the following equations, but not by both:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \tag{24}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}. \tag{25}$$

Both equations describe a 2-port which, when terminated at either port in an impedance Z, provides at the other port an input impedance $-Z$. However, the two characterizations correspond to two different circuits. An example of the first type (24) is the circuit shown in Fig. 4(a). It is obvious that the only excitation mode (out of the possible idealized four) for which this circuit will provide single-valued response is that under which port 1 is CEVR and port 2 is VECR. Therefore the circuit of Fig. 4(a) is an oriented network and should be described by the unidirectional equation (24). An example of the second type of realization (25) is the circuit of Fig. 4(b).

Since the excitation mode $\mathcal{E}$ for each of the two types of realizations of the NIC is a mixed one, we expect at the outset that each of the two NICs will have asymmetrical stability characteristics at the two ports. It is also obvious that symmetrical stability behavior would be possible only if an NIC can exist that has a single-valued response under a symmetrical idealized excitation mode. This in turn only can happen if an ideal NIC can be described by Y or Z parameters, a feat which is admitted to be impossible.

Let us now proceed to find the feedback model of the NIC of (24). If this NIC is fed at port 1 with a voltage source V_s having a source resistance R_1 and terminated at port 2 in a resistance R_2, the parameters of the feedback model will be

$$\boldsymbol{G} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 1/R_1 & 0 \\ 0 & R_2 \end{bmatrix}$$

for which

$$\det(\mathbf{1} + \boldsymbol{GB}) = 1 - R_2/R_1.$$

Therefore, for stable operation, condition (23) implies that

$$1 - R_2/R_1 > 0, \qquad \text{i.e., } R_1 > R_2.$$

A similar analysis for the NIC of (25) will reveal that for stable performance $R_1 < R_2$. In case that $\boldsymbol{G}$ and/or $\boldsymbol{B}$ is a function of p, condition (22) should be used. Then the obtained results will agree with those obtained by other techniques. However, the analysis of the ideal case cannot be obtained by any other method known to the authors.

At this point a comment is due on the proof by Davies [2] concerning the possibility of constructing an NIC with symmetrical stability behavior. Although the proof is mathematically sound, it does not represent a physical situation. The characterization given corresponds to the equivalent circuit shown in Fig. 4(c). To study the stability, an expression is derived for the input impedance of the device at port 1 when port 2 is terminated in an impedance $Z_2(p)$. The zeros and poles of the input impedance are then used to determine the short-circuit and open-circuit stability, respectively. This corresponds to studying the network behavior when excited at port 1 by an ideal voltage source in the first case and by an ideal current source in the second case. However, from the equivalent circuit of Fig. 4(c) it is apparent that under current excitation at port 1, the voltage V_1 is indeterminate and the expression for the input impedance is meaningless.

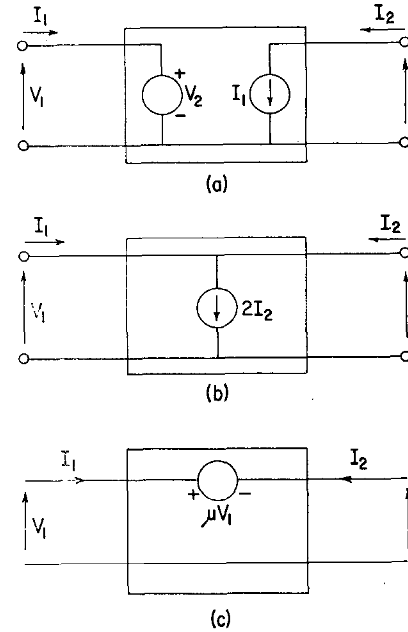


Fig. 4. Some NIC implementations.

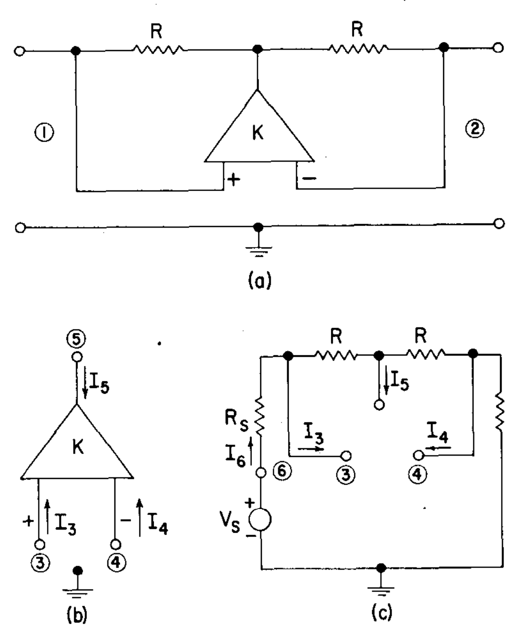


Fig. 5. (a) Particular NIC implementation. (b), (c) Derivation of feedback model for (a).

Any practical implementation of the model in Fig. 4(c) cannot operate under this mode of excitation.

It is interesting to note that the hypothesis made by Davies about the existence of an active impedance, stable both under short-circuit and open-circuit conditions and exhibiting a negative real part at a particular frequency, bears similarity to the postulation of the existence of a 1-port negative resistance characteristic similar to that shown in Fig. 1(a). Such a device represents a nonoriented network and therefore does not have a feedback model. However, such characteristics do not physically exist to the best of the authors' knowledge.

A Particular NIC: Another interesting example that demonstrates the power of the present approach is the study of the stability of the circuit shown in Fig. 5(a). This circuit will behave as an ideal NIC when the operational-amplifier gain approaches infinity. Contrasted with the NICs shown in Fig. 4, the present circuit looks symmetrical, and the orientation of the two ports is not obvious.

Let it be required to study the stability of the network when excited with a voltage source V_s in series with a resistance R_s at port 1 and terminated in a resistance R_L at port 2. Because of the lack of orientation information at ports 1 and 2, we look for a part of the network that is oriented. The operational amplifier offers such a part $\mathcal{N}$. It is an oriented 3-port [Fig. 5(b)] having $\mathcal{E}=(v_3, v_4, i_5)$. The rest of the network including the termination can now be considered as a coupling network $\hat{\mathcal{N}}$ to the source V_s. If the operational amplifier is assumed to have a gain K, the defining relations for $\mathcal{N}$ and $\hat{\mathcal{N}}$ can be written as

$$\begin{bmatrix} I_3 \\ I_4 \\ V_5 \end{bmatrix} := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ K & -K & 0 \end{bmatrix} \begin{bmatrix} V_3 \\ V_4 \\ I_5 \end{bmatrix}$$

$$\begin{bmatrix} V_3 \\ V_4 \\ I_5 \\ I_6 \end{bmatrix} := \begin{bmatrix} \dfrac{-RR_s}{R+R_s} & 0 & \dfrac{R_s}{R+R_s} & \dfrac{R}{R+R_s} \\ 0 & \dfrac{-RR_L}{R+R_L} & \dfrac{R_L}{R+R_L} & 0 \\ \dfrac{-R_s}{R+R_s} & \dfrac{-R_L}{R+R_L} & -\left[\dfrac{1}{R+R_s}+\dfrac{1}{R+R_L}\right] & \dfrac{1}{R+R_s} \\ \dfrac{R}{R+R_s} & 0 & \dfrac{-1}{R+R_s} & \dfrac{1}{R+R_s} \end{bmatrix} \begin{bmatrix} I_3 \\ I_4 \\ V_5 \\ V_6 \end{bmatrix}.$$

Using (15) and (18),

$$GB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ K & -K & 0 \end{bmatrix} \cdot \begin{bmatrix} \dfrac{RR_s}{R+R_s} & 0 & \dfrac{-R_s}{R+R_s} \\ 0 & \dfrac{RR_L}{R+R_L} & \dfrac{-R_L}{R+R_L} \\ \dfrac{R_s}{R+R_s} & \dfrac{R_L}{R+R_L} & \left[\dfrac{1}{R+R_s}+\dfrac{1}{R+R_L}\right] \end{bmatrix}.$$

Thus

$$\det(1+GB) = 1 + K\left(\frac{R_L}{R+R_L} - \frac{R_s}{R+R_s}\right).$$

Applying condition (23) yields for stable operation

$$\lim_{K\to\infty}\left[1 + K\left(\frac{R_L}{R+R_L} - \frac{R_s}{R+R_s}\right)\right] > 0$$

i.e.,

$$\frac{R_L}{R+R_L} - \frac{R_s}{R+R_L} > 0$$

or

$$R_L > R_s.$$

C. Negative Impedance Inverters

The NIV can be described in terms of either impedance or admittance parameters [10]. Fig. 6 shows two possible controlled source circuit realizations for the NIV. It is obvious that the NIV of Fig. 6(a) is an oriented network with $\mathcal{E}=(i_1, i_2)$. Thus it can be described by

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} := \begin{bmatrix} 0 & R \\ R & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}.$$

If this NIV realization is fed at port (1) with a voltage source V_s in series with a resistance R_1 and terminated at port 2 in a resistance R_2, the feedback model will be given by

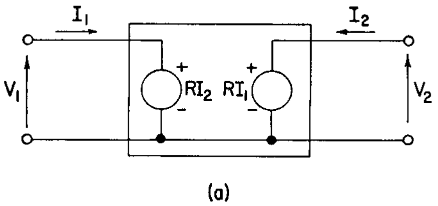
$$G = \begin{bmatrix} 0 & R \\ R & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1/R_1 & 0 \\ 0 & 1/R_2 \end{bmatrix}$$

for which

$$\det(1+GB) = 1 - \frac{R^2}{R_1R_2}$$

and application of (23) yields the condition for stable opera-



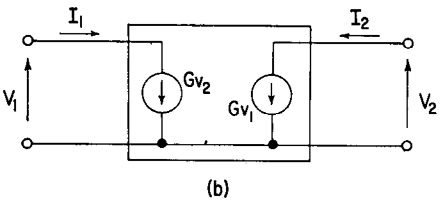


Fig. 6. Two dual implementations of NIV.

tion as

$$R_1 > R^2/R_2.$$

For the dual NIV of Fig. 6(b) the condition for stable operation can be shown to be

$$R_1 < R^2/R_2.$$

Each of the preceding conditions indicates the symmetrical stability behavior of the two ports of an NIV. This corresponds to the fact that the ideal NIV can be described by a Z or a Y matrix.

VI. Conclusion

The concept of network orientation is suggested to be a vehicle enabling the derivation of a feedback model to represent any active network. Through this model many of the methods and techniques developed for the study of feedback systems can be directly applied to active networks.

The technique has been used to study the stability of negative resistance circuits. It has been shown that the stability behavior of these circuits can be systematically obtained without explicit description of the parasitic elements.

The method presented should be regarded as a practical rather than a mathematically rigorous approach.

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