Fast Amplitude Stabilization of an *RC* Oscillator

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***Abstract*—A technique is proposed for stabilizing the output amplitude of a variable-frequency *RC* sine-wave oscillator. Amplitude settling time is reduced by converting amplitude to a dc voltage having only a small ripple content, which requires little filtering. A relationship between amplitude transient response and harmonic distortion is demonstrated, and the results are compared to those obtained by more conventional methods.**

IT IS an unfortunate fact that a stable sine-wave oscillator cannot be constructed entirely from linear elements, since total linearity would leave the output amplitude indeterminate. Some form of nonlinearity, then, must be added to regulate the output level. One common regulation scheme employs nonlinear bounding, as by amplifier saturation or by added zener diodes. Another uses a two-terminal voltage-dependent resistor with a built-in time constant, such as the lamp or thermistor used in the classical Wien-bridge oscillator. A third technique incorporates voltage-controlled attenuators using FET's or analog multipliers [1]. The last of these methods allows convenient control of loop gain and time constants, and thus has the greatest potential for good amplitude regulation independent of ambient temperature variations and changes in the frequency of oscillation, but presents one major problem. The dc input to the voltage-controlled attenuator must have very little ripple or else excessive distortion of the output waveform results. If this dc voltage is derived from the usual half-wave rectifier, an extremely long filter time constant is required. The oscillator output level consequently settles very slowly after a disturbance such as that which results from a change in frequency. This behavior is undesirable in low-frequency applications, particularly when the frequency must be changed often.

Very few solutions to these problems are proposed in the literature. Freshour [2] uses a combination of nonlinear bounding and a voltage-controlled attenuator to gain some control over the shape of the amplitude transient response, although settling times are still fairly long. Meyer-Ebrecht [3] has suggested the use of a sample-hold in the control loop. Although this method certainly leads to low distortion figures, it is not very practical except at low frequencies, since the sampling interval must be a very small part of a cycle. This precludes its use in conventional wide-range oscillators, except perhaps at considerable expense.

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The solution proposed here reduces settling time considerably by replacing the half-wave rectifier by one whose output at any instant is the most positive of four inputs spaced 90° apart in phase, all of which are derived from the oscillator. The output spectrum for such a rectifier is shown alongside that of the half-wave version in Fig. 1.

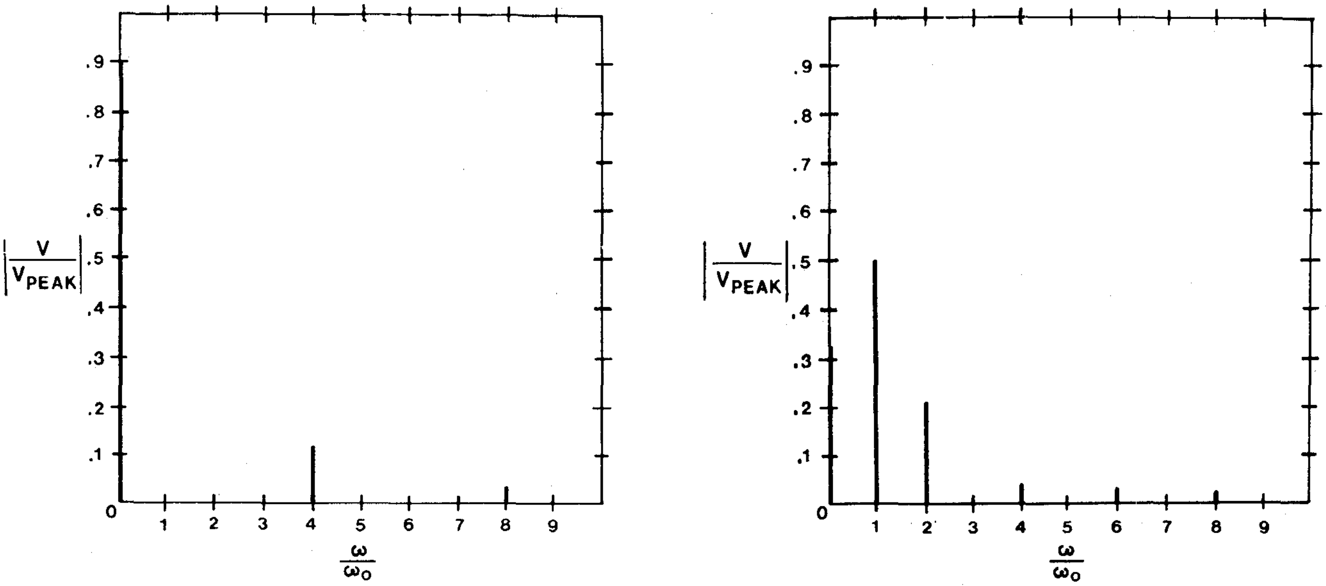
A complete oscillator employing this method of amplitude detection is illustrated in Fig. 2. The oscillator itself is a cascade of two integrators A_1 and A_2 with an inverter A_3. This arrangement is really a state-variable filter configuration designed for infinite Q[4]. Inverter A_4 supplies the fourth phase to the rectifier, and a current-output analog multiplier provides a small amount of feedback of either sign as a means of controlling amplitude. The magnitude and sign of this feedback are determined by the dc control input to the multiplier. This control voltage is in turn derived from the difference between the rectifier output and a temperature-compensated reference voltage. This difference appears at the output of A_5, and is applied to the multiplier along with its integral from A_6. Triple potentiometer R_1 varies the frequency of oscillation over a decade, and capacitors C_1, C_2, and C_3 can be switched for range changing. The component values shown are for 100 Hz to 1 kHz. The amplifiers used are LM301A's, which are suitable up to about 100 kHz with feedforward frequency compensation [5]. The multiplier is an MC1594 [6].

The response of the stabilization loop could be predicted quite accurately by treating the rectifier as a switch which samples the four rectifier inputs sequentially for one quarter-cycle each. Such an analysis would be quite tedious, however, and would probably not yield much insight into the problem. A much simpler analysis is appropriate if the distance from the closed-loop stabilization poles to the origin is much less than ω_0. Then it is possible to consider only average behavior over a cycle (that is, treat the rectifier output as a continuous amplitude signal), and thus derive a linear, continuous model which represents approximately the response of oscillator output to multiplier control voltage.

First, note that the multiplier can be regarded as a transconductance

$$y_{21} = \frac{I_o}{V_1} = K_m V_c \tag{1}$$

where I_o and V_1 are the peak multiplier output current and peak signal input voltage, respectively, and K_m is the multiplier scale factor. The characteristic equation for the oscillator itself (disregarding the amplitude control loop) is



(a) (b)

Fig. 1. (a) Four-phase rectifier output spectrum.

$$\frac{4\sqrt{2}\,V_{\text{Peak}}}{\pi}\left[\frac{1}{2}+\frac{1}{15}\cos 4\omega_0 t-\frac{1}{63}\cos 8\omega_0 t+\cdots\right].$$

(b) Half-wave rectifier output spectrum.

$$V_{\text{Peak}}\left[\frac{1}{\pi}+\frac{1}{2}\sin\omega_0 t-\frac{2}{3\pi}\cos 2\omega_0 t-\frac{2}{15\pi}\cos 4\omega_0 t-\cdots\right].$$

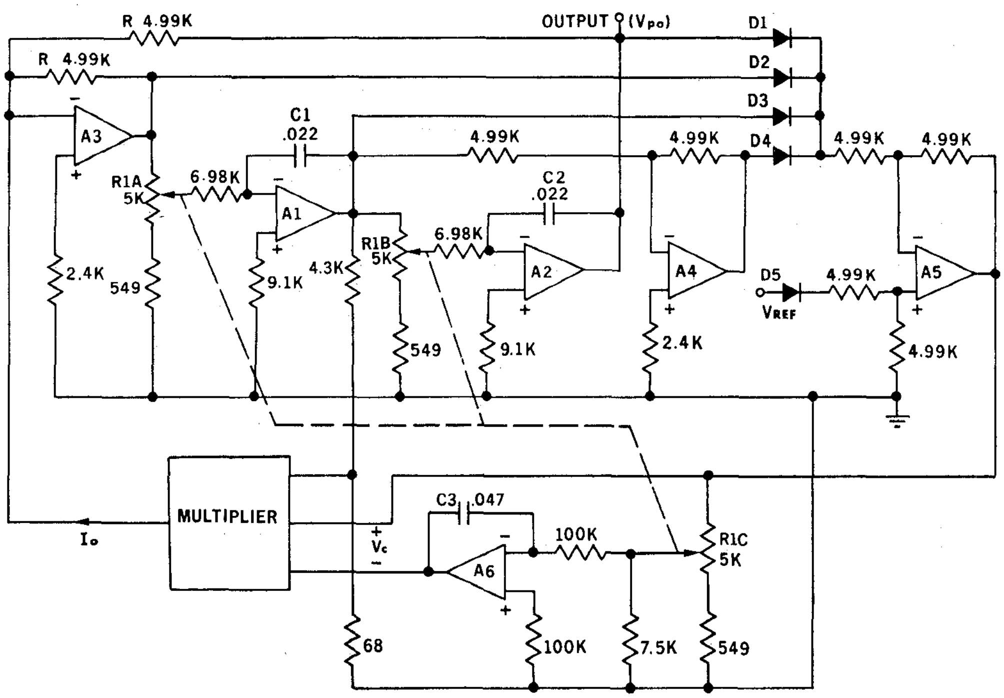


Fig. 2. Oscillator using the four-phase rectifier.

$$s^2 - y_{21}R\omega_0 s + \omega_0^2 = 0. \tag{2}$$

This equation has a solution of the form

$$V = V_{po}\exp\left(y_{21}R\omega_0 t/2\right)\cos\omega_0 t \tag{3}$$

where V_{po} is the initial peak output voltage. Only the amplitude is of interest. The amplitude is given by

$$V_p = V_{po}\exp\left(y_{21}R\omega_0 t/2\right). \tag{4}$$

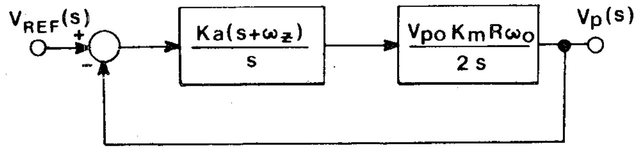


Fig. 3. Block diagram of the amplitude regulation loop.

The exponent $y_{21}R\omega_0 t/2$ can be assumed to be small if only small amplitude disturbances are to be considered ($|V_{po} - V| \ll V_{po}$). In this case, the exponential can be approximated by the first two terms of its Taylor series:

$$V_p \approx V_{po}[1 + (y_{21}R\omega_0 t/2)]. \tag{5}$$

From (1) and (5),

$$V_p \approx V_{po} + V_c t\left(\frac{K_m V_{po} R\omega_0}{2}\right). \tag{6}$$

Note that the response of output amplitude to a step of multiplier control voltage is that of an integrator whose gain constant is $V_{po}K_m R\omega_0/2$.

The remainder of the control loop is straightforward. An additional integration must be included if the steady-state amplitude error is to be zero, and this additional integration necessitates the addition of a zero for stability. This zero is provided by feeding the error directly from the rectifier to the multiplier, along with the integrated error. Another pole could be added for ripple filtering, but only at the cost of a considerable increase in complexity.

The complete open-loop transfer function is

$$G(s) = \left(\frac{V_{po}K_m R\omega_0}{2s}\right)\left[\frac{K_a(s + \omega_z)}{s}\right]. \tag{7}$$

A block diagram is shown in Fig. 3. For the component values shown in Fig. 2, the error integrator gain $\omega_z = (0.047\ \mu F \times 100\ \mathrm{k}\Omega)^{-1} = 213$ and K_a is 0.90, which is the ratio of dc rectifier output voltage to peak input voltage.

The loop can be optimized by selecting K_aK_m such that, for a particular value of ω_z, the product of total harmonic distortion and settling time is minimum. Let T_s represent the time required for the output to settle within 5 percent after a step change in reference voltage. It is apparent from the root locus of Fig. 4 that as K_aK_m increases, the closed-loop poles move to the left around the circle and T_s decreases. If T_s is large compared to the period of oscillation, ω_z will be small compared to $4\omega_0$, and therefore the ripple voltage which reaches the multiplier input will come almost entirely from the proportional feedback rather than from the integrator. For a given value of ω_z, the multiplier output current due to ripple is proportional to K_aK_m. Since harmonic distortion is proportional to this current, the optimum value of K_aK_m is that which minimizes $K_aK_mT_s$. Computation of this product for a wide range of values of K_aK_m has shown that a minimum exists for

$$\frac{V_{po}K_aK_m R\omega_0}{2} = \omega_z. \tag{8}$$

For this value of gain, the poles are located as shown in

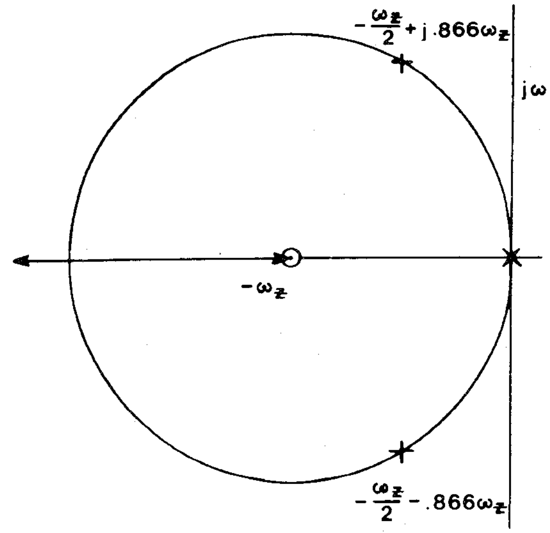


Fig. 4. Root locus for the loop transfer function.

Fig. 4. The envelope overshoots about 30 percent, then falls below the final value by just less than 5 percent. The settling time is given by

$$T_s = \frac{4.378}{\omega_z}. \tag{9}$$

Normalizing with respect to the period of oscillation gives

$$\frac{T_s}{T} = \frac{4.378\omega_0}{2\pi\omega_z} = \frac{0.698\omega_0}{\omega_z}. \tag{10}$$

It should be noted that if $T_s = T$, $4\omega_0 = 5.7\omega_z$, and the ripple in the integrator output is still small.

Referring to Figs. 1 and 2, the multiplier output current is

$$I_o = -\frac{K_aK_m 4\sqrt{2}\ V_{po}^2}{0.9\pi}\cos\omega_0 t \cdot\left(\frac{1}{15}\cos\omega_0 t - \frac{1}{63}\cos 8\omega_0 t + \cdots\right). \tag{11}$$

Combining (8), (10), and (11) gives

$$I_o = -\frac{17.51\sqrt{2}\ V_{po}}{0.9\pi^2 R(T_s/T)}\cos\omega_0 t \cdot\left(\frac{1}{15}\cos 4\omega_0 t - \frac{1}{63}\cos 8\omega_0 t + \cdots\right). \tag{12}$$

Application of a familiar trigonometric identity yields

$$I_o = -\frac{8.757\sqrt{2}\ V_{po}}{0.9\pi^2 R(T_s/T)}\left[\frac{1}{15}(\cos 3\omega_0 t + \cos 5\omega_0 t) - \frac{1}{63}(\cos 7\omega_0 t + \cos 9\omega_0 t) + \cdots\right]. \tag{13}$$

A simple analysis shows that the transfer function from I_o to V_o is that of a second-order low-pass filter,

$$\frac{V_o(s)}{I_o(s)} = -\frac{R\omega_0^2}{s^2 + \omega_0^2}. \tag{14}$$

The amplitudes of individual harmonics at the output can be computed conveniently by defining

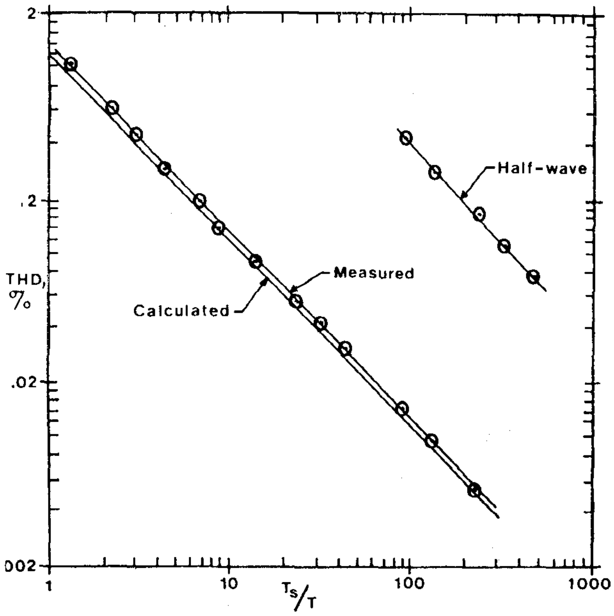


Fig. 5. Measured and calculated harmonic distortion versus settling time.

$$n = \frac{\omega}{\omega_0}. \quad (15)$$

Then the ratio of output voltage at the nth harmonic to the component of I_o at that frequency is

$$\left|\frac{V_{on}}{I_{on}}\right| = \frac{R}{n^2 - 1}. \quad (16)$$

Finally, the oscillator output due to the injection of harmonics into the multiplier may be found from (13) and (16) to be

$$|V_{on}| = \frac{8.757\sqrt{2}\, V_{po}}{0.9\pi^2(T_s/T)} \left[\frac{1}{15}\left(\frac{\cos 3\omega_0 t}{8} + \frac{\cos 5\omega_0 t}{24}\right) - \frac{1}{63}\left(\frac{\cos 7\omega_0 t}{48} + \frac{\cos 9\omega_0 t}{80}\right) + \cdots\right]. \quad (17)$$

The third and fifth harmonics clearly predominate. Third-harmonic distortion is given in percent by

$$100 \times \left|\frac{V_{o3}}{V_{po}}\right| = \frac{72.97\sqrt{2}}{9\pi^2(T_s/T)} \quad (18)$$

and fifth-harmonic distortion is

$$100 \times \left|\frac{V_{o5}}{V_{po}}\right| = \frac{72.97\sqrt{2}}{27\pi^2(T_s/T)}. \quad (19)$$

Ignoring higher order terms, the total distortion is

$$THD = \frac{72.97\sqrt{2}}{9\pi^2(T_s/T)}\sqrt{1 + \frac{1}{9}} \approx \frac{1.22}{T_s/T}. \quad (20)$$

This equation is plotted in Fig. 5, along with measured results. Also shown are values measured with a half-wave rectifier (obtained by removing D_2, D_3, and D_4 of Fig. 2, and replacing the reference voltage by a current reference applied to the summing junction of A_6).

It is apparent that the behavior of the oscillator and four-phase rectifier is as predicted. It can also be seen that, for the same level of harmonic distortion, the four-phase rectifier allows the oscillator to settle about 30 times faster than with a half-wave rectifier. This improvement is certainly significant in low-frequency applications, say a few hertz, where an operator might normally have to wait tens of seconds before making a measurement. Its fast-settling characteristic makes the oscillator ideal for automated frequency response testing. Control by an external digital or analog signal is possible with only minor modifications.

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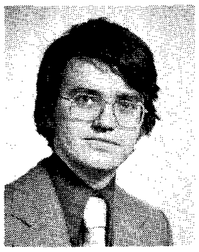

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