M3.26

The Angle-of-Sight Signature for 2D Shape Analysis

*I. Tchoukanov** — Research Assistant
R. Safaee-Rad+ — Ph.D. Candidate
B. Benhabib+ — Assistant Professor
K.C. Smith+ #* — Professor

Computer Integrated Manufacturing Laboratory, Department of Mechanical Engineering
University of Toronto, 5 King's College Road, Toronto, Ontario, Canada M5S 1A4
+ Department of Mechanical Engineering * Department of Electrical Engineering # Department of Computer Science

ABSTRACT

In this paper, we introduce a new 2D shape-encoding scheme, which is based on the idea of the *angle-of-sight*. The shape can be efficiently transformed into 1D signature by recording the angle-of-sight-vs-distance of each boundary point with respect to a shape-specific chord-of-sight. The chord-of-sight is selected by using an extension of the notion of shape to geometrical entities, which are not part of the shape boundary -- to the idea of shape-specific points and the characteristic ellipse. The angle-of-sight signature is a unique, information-preserving, transformation-invariant shape representation. It is demonstrated that the angle-of-sight shape-encoding scheme is capable of deriving well-behaved, noise-tolerant signals, convenient for shape matching through alignment of signatures.

1. Introduction

Shape representation and matching is a key problem in machine-vision system development. We face this problem in the context of a new active-object-recognition system [1]. The main design concept is to reduce the dimensionality of the recognition task. A 3D object is modeled in this system by using a small set of distinct perspective views, called "standard-views". Shape matching is performed between the acquired 2D "standard-view" of the sensed object with unknown identity and a library of 2D "standard-views" of a set of objects.

Following the same concept of dimensionality reduction in a "top-to-down" fashion, we address the problem of invariant shape-encoding-scheme design to transform the 2D "standard-views" into 1D signatures suitable for signal matching.

A number of shape-representation techniques have been reported in the literature [2], [3]. A signature is defined as a 1D signal derived from the shape by using an encoding scheme for mapping the information from the "shape" space to the "signature" space.

The "goodness" of a signature is admittedly related to its uniqueness, reliability, suitability for matching and low processing time/memory requirements [4].

The main contribution of this paper is the development of a new 2D-shape-encoding scheme. It is based on a new concept, -- the angle-of-sight -- for extracting and coding "shape" information.

2. Description of the angle-of-sight shape-encoding scheme

The angles of a triangle formed by any three non-collinear boundary points A, B and C (see Figure 1) are invariant under shape translation, rotation and scaling. To imagine how to make this property functional for a shape-encoding scheme, we can perform the following thought experiment: predetermine two of the vertices, say A and B, and keep them fixed while moving the third vertex C along the boundary (see Figure 1). Then, the angle α, associated with each position of the moving vertex C, is a descriptive property of the boundary and can be used for shape representation. Hereafter, the chord joining the fixed points is referred to as the chord-of-sight (COS) and the angle α at the third vertex is referred to as the angle-of-sight (AOS).

The AOS signature is a boundary-based descriptor of a planar curve and is defined as a 1D signal AOS=f(l), where l is the arc length between a starting point C_0 and the boundary tracing point C in the clockwise direction. The AOS function is defined as follows:

$$f(l) = \arccos\left(\frac{d_1^2(l) + d_2^2(l) - d^2}{2d_1(l)d_2(l)}\right), \quad f(l) \subseteq (0, \pi) \quad (1)$$

where $d_1(l)$ and $d_2(l)$ are the distances from the tracing point to the end-points of the COS, and d is the length of the COS.

So defined, the AOS signature of a planar curve with unit length has the following properties:

- It is transformation invariant;
- It is a periodic signal if the boundary is closed;

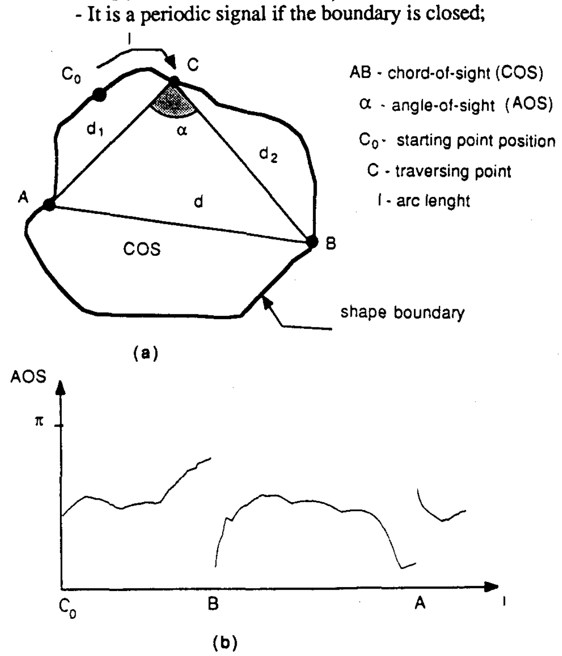


Figure 1(a) Definition of the Angle-of-Sight Scheme
(b) The Derived Signature

CH2977-7/91/0000-2461 $1.00 © 1991 IEEE

- Change of the starting point position causes only a cyclic shift of the signature;
- It is computationally inexpensive.

The AOS coding scheme uniquely defines an orthogonal object-specific coordinate system, which can be used to select a "standard" position of the starting point C_0. Let us consider the geometry of this situation using Figure 2. The loci of boundary points having one and the same AOS are circular arcs passing through the end-points of the COS. The locus of the centers of these constant AOS arcs is a straight line orthogonal to the COS and passing through its midpoint. The crossing point of this line and the boundary is an object-specific starting point for the AOS signature.

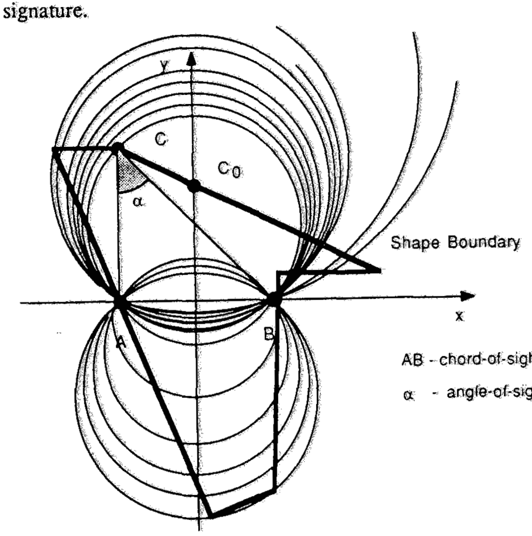


Figure 2. The AOS-Related Coordinate System

3. Chord-of-sight selection requirements

The AOS coding scheme implies that the identity of the COS end-points is predetermined (by off-line selection), whereas their location is detected in real time. Obviously, the AOS signature of a shape is transformation-invariant only if one and the same pair of COS end-points is unambiguously detected for each position, orientation and size of the shape. As far as the shape-recognition problem is concerned, matching AOS signatures is to be performed only if they are derived with reference to a COS with one and the same identity. Therefore a COS-selection process must be established. It can be formulated as follows:

- Uniqueness. The COS end-points have to be uniquely identifiable from the shape boundary (or features derived from it);
- Commonality. The COS end-points have to rely on features common to a given set of shapes;
- Boundary-distortion tolerance. The COS end-points must be chosen so as to be minimally sensitive to boundary noise;
- Detectability. The COS end-points have to be identified fast and readily in real time.

The last requirement is in contradiction with the first ones, as can be seen: Apparently, the first three requirements can be satisfied by points having a large domain of support. Consequently, detection of such prominent points (in a pointwise sense) would require more computational time. Points relying on a local domain of support, such as corners, dominant points, etc., can hardly meet the above-stated requirements. Generally speaking, the end points of the longest boundary primitives, such as linear or circular segments, are better candidates for chord-of-sight, but their detection requires considerable processing time [5], [6].

Ideally, the best candidates for COS end-points are points to whose coordinates all boundary points collectively contribute.

4. Chord-of-sight based on shape-specific points

Shape-specific points are an extension of the notion of shape to include points which do not lie on the shape boundary. The formal definition of a shape-specific point is given in [7]. For the sake of clarity, it will be repeated here.

Definition: "Let p=F(S) be a point computed from shape S according to procedure F. Also, let S'=T(S) , where T is a planar transformation (translation, rotation or dilation), and let p'=F(S'). Then p is a shape-specific point of S with respect to transformation T if and only if p'=T(p)."

This definition also applies to geometrical entities other than points. For instance, the length of a chord connecting two shape-specific points possesses the property of being also shape-specific.

Shape-specific points have several properties which make them attractive to the AOS encoding scheme:

- Shape-specific points behave as if they were on the shape boundary;
- The coordinates of shape-specific points are computed rather than detected;
- All boundary points collectively contribute to the computation of the coordinates of shape-specific points. In this sense, the whole shape boundary acts as a region of support for the shape-specific points.

Evidently, an AOS signature derived with respect to a COS based on shape-specific points, retains the property of being shape-transformation-invariant. The method for signature-starting-point selection, given in section 2, holds for shape-specific COS end-points as well.

Various functions can be defined for shape-specific-point computation. Mitiche and Aggarwal in their work [7] implement the centroid and the weighted median point to recover shape orientation for the purpose of registration. Our experiments, however have indicated that boundary noise causes a significant shift in the starting point position of the corresponding AOS signature when the COS is based on these two points. This is explained by the geometry of Figure 3, where the COS end-points are shown affected by isotropic noise with standard deviation ε. The shift of the signature-starting-point position lies within an angular interval determined by the following formula:

$$\beta = 2 \arcsin(\frac{2\varepsilon}{d}) \qquad (2)$$

This formula suggests that the noise-induced shift of the AOS

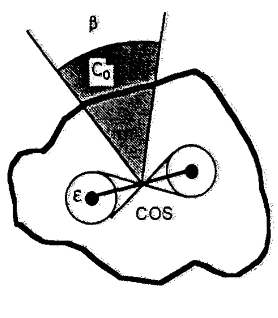


Figure 3. Noise-Induced Shift of the Signature Starting Point

signature starting point is minimized by increasing the length of the COS. The distance between the centroid and the weighted mean point of a compact shape usually is small, a fact which deteriorates the noise-tolerance of the corresponding AOS signature. In view of this consideration, better results can be achieved by using shape-specific points computed from a Fourier expansion of the shape boundary. Kuhl and Giardina [8] apply Fourier trigonometric expansion to the X and Y projections of a closed contour and show that the vectors corresponding to each harmonic have elliptical loci. The five basic parameters of the ellipse related to the fundamental Fourier harmonic are given in [9]. We have proved that this ellipse is shape-specific (the proof will be omitted due to lack of space).

In our opinion, this shape-specific ellipse is an important extension of the notion of shape: Hereafter, the shape-specific ellipse related to the first harmonic (the fundamental frequency) will be referred to as the characteristic ellipse (CE).

5. An Improved AOS coding scheme

The AOS function is not defined for boundary points which are collinear with the end-points of the chord-of-sight, and correspondingly, the signature suffers from having jump discontinuities for such points. Jump discontinuities hinder matching of signatures and deteriorate the spectral characteristics of the signal (through Gibb's phenomenon).

This problem can be solved by placing one of the end-points of the COS outside of the plane of the "picture". Let us consider an orthogonal XYZ frame (see Figure 4), where the origin is coincident with the center of the characteristic ellipse of the boundary, and the XY axes are aligned with its major and minor radii. The length of the COS is defined to be equal to the major radius of the characteristic ellipse of the boundary. In the same manner as in section 2, the AOS signature is defined as a 1D signal AOS=f(l), where l is the arc length between the moving point C and a starting point C_0, measured in a clockwise direction. Now the AOS function has no discontinuities and is defined as follows:

$$f(l) = \arctan\left(\frac{a}{r(l)}\right) \qquad f(l) \subseteq (0, \pi/2) \qquad (3)$$

where a is the length of the major axis of the characteristic ellipse and r(l) is the distance from a boundary point to its center.

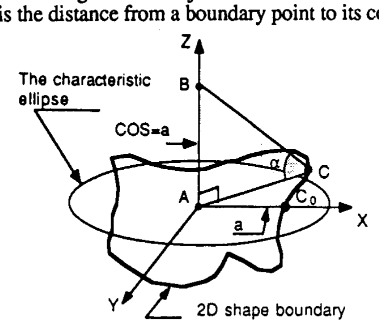


Figure 4. Definition of an Improved Angle-of-Sight Scheme

The crossing point of the boundary and the Y-axis is used as a "standard position" for the starting point. Kuhl and Giardina [8] use the same approach to compute a normalized Fourier descriptor.

It can be seen that this improved version of the AOS encoding scheme has interesting properties:

- It provides an inherent smoothing of the shape representation because the transfer function (y=arctan(x)) is always under the line y=x. This smoothing is more pronounced for longer chord-of-sights, and has a potential for boundary-noise reduction. A family of AOS signatures of a shape presented in Figure 5 shows that longer COS cause smoothing of the form of the signal and an increase of its DC component;

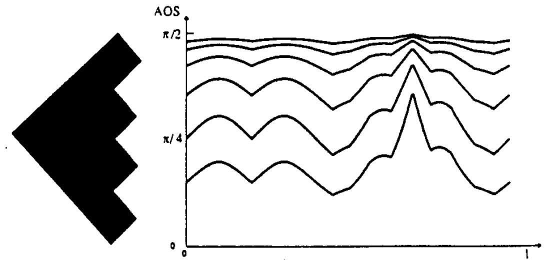


Figure 5. A Shape and the Corresponding Family of AOS Signatures for Various Lengths of the COS -- 0.5a, a, 2a, 4a, 8a, 16a

- The signature preserves shape symmetry in the sense that points equidistant from the centroid have an equal AOS;
- The simplest possible signature --a straight line -- belongs to the circle -- the basic shape. In this sense, the AOS signature shows how the shape deviates from the basic shape;
- The signature preserves shape information. Signatures from which the shape can be recovered are referred to as information-preserving signatures [2]. To prove this property, it is sufficient to show a procedure performing "the shape-from-representation" transform. Figure 6(a) shows the geometry of such a shape-recovery transform. The AOS signature is defined by a discrete sequence AOS(i)={f(l_i)}, i=0,1,... N-1, parameterized with respect to the arc length. The 3D locus of points having constant angle-of-sight is the surface of a toroid (a surface of revolution generated by rotating constant-AOS arcs around the chord-of-sight -- see Figure 6 (b)). The intersection of the i-th AOS toroidal locus with the picture plane is a circle (see also Figure 4), with radius:

$$R_i = \frac{a}{2\sin(AOS(i))}, \qquad (4)$$

where a is length of the chord-of-sight. Consequently, a shape-recovery procedure can be described as follows:

1. For i=0 to N-1 draw concentric circles with radius R_i;
2. Choose an arbitrary starting point P_0, lying on the first circle (R_0);
3. For i=1 to N-1 draw a circle centered at point P_{i-1} with radius $l=l_i - l_{i-1}$. The boundary points P_i are recovered by taking the crossing points of this circle and circle (R_i) in the direction of traversal.

Thus, the existence of a unique inverse transform proves that the AOS shape-encoding scheme is an information-

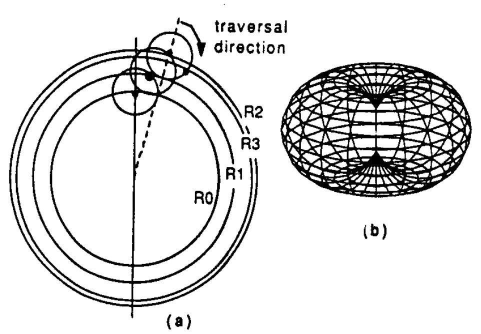


Figure 6(a) Shape Recovery from the Improved AOS Signature (b) 3D Locus of Constant AOS Points

preserving, one-to-one mapping. Due to its uniqueness, the AOS signature is appropriate for shape representation and matching.

6. Matching of AOS signatures

We have shown in section 4 that the locus of the shape boundary projections related to the fundamental frequency Fourier harmonic is a shape-specific ellipse, which behaves as an inseparable part of the shape. Figuratively speaking, each shape is "accompanied" by such a characteristic ellipse. This idea suggests that two shapes can be matched by first aligning their characteristic ellipses. This process of alignment in the "picture" plane can be performed in two steps: First, superimpose the two shapes so that their centroids coincide, and subsequently, rotate one of the shapes until the major/minor axes of their ellipses align. Alignment of shapes in the "picture" plane is equivalent to superimposing their AOS signatures: Recall that the AOS signature is transformation-invariant, and that its starting point lies on the major axis of the characteristic ellipse.

Conceptually, if two shapes are similar, a dissimilarity measure of their signatures is expected to achieve its global minimum within a small window ∇ centered around the starting point of the superimposed signatures. This window "shift-and-match" search is necessary to allow for possible noise-induced displacement of the characteristic ellipse. We use the following average pointwise dissimilarity measure:

$$D(j) = \frac{1}{K}\sum_{i=0}^{K-1} (S_1(i+j) - S_2(i+j))^2 \qquad j \subseteq [-\nabla, \nabla]. \quad (5)$$

where $S_1(i)$, $i=0,1,\ldots M-1$ and $S_2(i)$, $i=0,1,\ldots N-1$ are two shape signatures and $K = \min(M, N)$.

The performance of the AOS dissimilarity measure is shown in Table 1. The test shapes -- rotated, scaled and distorted versions of a rectangular object -- are matched against a set of library objects The results obtained for the dissimilarity measure show that despite the fact that most of the library objects are quite similar, the test shapes can be identified using a simple minimum-distance rule. These results also indicate that, due to the uniqueness of the signature, geometrically dissimilar objects are classified as different.

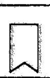
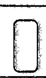
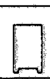
library / test shapes						
	4.78	12.63	14.06	9.35	94.74	13.82
	0.96	9.48	15.53	3.55	114.82	11.4
	0.125	9.29	18.13	2.26	122.58	13.87

Table 1. Distance Measure Performance of the Improved AOS Signature

If the eccentricity of the characteristic ellipse of a shape is below a certain threshold, the shape can be considered rotationally symmetrical. The boundary of a rotationally symmetric shape is a sequence of periodically repeating group. The period of repetition is determined by the order of the shape symmetry. Algorithms for symmetry detection are given in a number of papers (see for example [10]). Once the order of rotational symmetry is detected, only that part of the signature which corresponds to the periodic boundary group, is used in the process of matching. The problem of starting-point correspondence vanishes because the signatures are partial and therefore open.

7. Conclusions

In this paper we addressed the topic of invariant shape-encoding scheme for converting 2D planar shapes into 1D signals. According to the new scheme, developed here, the "shape" information is encoded in terms of "angle-of-sight" information derived for each boundary point with respect to a shape-specific "chord-of-sight". We have demonstrated that the AOS signature is a unique, information-preserving representation, which is independent of the position, orientation and size of the shape. Besides, unlike many other signatures, the AOS signature does not have jump discontinuities. These properties of the AOS signature make it appropriate for shape description and matching and for signal matching through alignment in particular. The results obtained for a particular signature dissimilarity measure indicate that the AOS representation is quite robust under shape distortion.

ACKNOWLEDGEMENT

This work was partially supported by the Manufacturing Research Corporation of Ontario (MRCO).

References

[1] Safaee-Rad, R., Benhabib, B., Smith, K.C., and Zhou, Z., "Pre-Marking Methods for 3D Object Recognition", *IEEE, Proc., Int. Conference on System, Man and Cybernetics*, pp.592-595, Boston, MA, Nov. 1989.

[2] Pavlidis, T., "Algorithms for Shape Analysis of Contours and Waveforms", *IEEE, Trans. on Pattern Analysis and Machine Intelligence*, Vol. PAMI-2, No. 4, pp. 301-312, July 1980.

[3] Marshal, S., "Review of Shape Coding Techniques", *Image and Vision Computing*, Vol. 7, No. 4, pp.281-294, Nov. 1989.

[4] O'Rourke, J., and Washington, R.,"Curve Similarity via Signatures", In: *COMPUTATIONAL GEOMETRY*, Elsevier Science Publishers B. V (North-Holland), pp. 295-317, 1985.

[5] Ballard, D.H., "Generalyzing the Hough Transform to Detect Arbitrary Shapes", *Pattern Recognition* , Vol.13, No. 2, pp.111-122, 1981.

[6] Liao, Y., "A Two-Stage Method for Fitting Conic Arcs and Straight-Line Segments to Digitized Contours", *IEEE, Proc., Int, Conference on Pattern Recognition and Image Processing*, pp. 224-229, 1981.

[7] Mitiche, A., and Aggarwal, J.K., "Contour Registration and Shape-Specific Points for Shape Matching", *Computer Vision, Graphics and Image Processing*, Vol. 22, pp. 396-408, 1983.

[8] Kuhl, F. P., and Giardina, C. R., "Elliptic Fourier Features of a Closed Contour", **Computer Graphics and Image Processing 18**, pp. 236-258, 1982

[9] Safaee-Rad, R., Smith, K. C., Benhabib, B., and Tchoukanov I. K., "Application of Moments and Fourier Descriptors to the Accurate Estimation of Elliptical Shape Parameters", *IEEE, Proc., Int. Conference on Acoustics, Speech and Signal Processing*, 1991 (In print).

[10] Leou, J. and Tsai, W., "Automatic Rotational Symmetry Determination for Shape Analysis", *Pattern Recognition*, Vol. 20, No. 6, pp. 571-582, 1987.