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# TWO-DIMENSIONAL SHAPE REPRESENTATION USING MORPHOLOGICAL CORRELATION FUNCTIONS

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## ABSTRACT

This paper introduces a new descriptor for the representation of two-dimensional continuous or discrete signals. The proposed shape descriptor, which we call the *geometrical correlation function* (GCF), is based on the principle of mathematical morphology. The properties of this shape descriptor are examined. It is shown that the family of GCFs associated with different orientations of a particular shape is both translation, scale, and rotation invariant. In addition, geometrical properties such as the area and perimeter of the shape can be derived from the GCF family. The utilization of the GCFs for shape-recognition is then considered. The GCF family can be computed using the associated *morphological correlator* which is composed of $m$ parallel computation units and a feature-function selection unit where a small subset of the GCF family is selected for classification. It is shown that with a suitable criterion for selecting the feature function, promising results for successful classification are obtained.

## 1. INTRODUCTION

The problem of shape identification is an important one in pattern recognition which has received considerable attention in recent years. In particular, many applications in the area of automated or flexible manufacturing require high-speed object recognition as well as the identification of position, orientation and velocity of moving objects or machine parts.

There are basically two major steps involved in shape recognition, namely feature extraction and shape classification. The selection of meaningful features has been traditionally based on the structures used by human beings in interpreting pictorial information [1]. Given that a shape descriptor has been chosen, the first step is then to represent the unknown shape using this shape descriptor. This shape descriptor should be efficient in its use of computation, as well as faithful in providing a usable representation of the original shape. Though there exist many quantitative measures of shape, there is yet, in general, no agreement on a minimum set of shape descriptors to adequately quantify object form. Once a feature set is established, the second step in the recognition process is the classification of the unknown shape based on the creation of a suitable measure. This usually involves the application of some criterion such as Minimum Mean Square Error (MMSE).

Recently, the use of morphological techniques [2,3] for image analysis has become very popular as a consequence of the simple implementations corresponding to many morphological operations. Examples of shape representations based on morphological operations include the pecstrum [4] and the morphological skeleton [5].

In this paper, a new 2-D shape descriptor, namely the *geometrical correlation function* (GCF), is proposed and its applications in shape recognition are demonstrated. In section 2, the definition and properties of the GCF are presented. Section 3 then describes a shape-recognition system, which utilizes the proposed shape descriptor and employs an associated *morphological correlator*. Finally, in section 4, the performance of the proposed scheme is analyzed using some experimental examples.

## 2. GEOMETRICAL CORRELATION FUNCTIONS (GCFs)

In this section, the mathematical definition and properties of the *geometrical correlation function*, which is closely related to the morphological covariance as defined in [6], are given. The morphological covariance basically descends from the theory of random functions of order two, and it provides a measure on the correlation properties of 2-D signals.

Specifically, the morphological covariance is defined as the measure of the eroded set by the structuring element $B$ which consists of a pair of points separated by a distance equivalent to the amount of spatial shift $h$, at an angle $\phi$.

Let $X$ be a deterministic compact set in $\mathbf{R}^n$, and $B^\phi$ be a structuring element which is composed of two single points separated by a distance $h$ at an angle $\phi$ relative to 0°. The morphological covariance $C(h)$ is defined as the measure of the set $X \ominus B^\phi$, which is $X$ eroded by $B^\phi$. If we consider the translation $B_x^\phi$ of $B$ by $x$ at an angle $\phi$, the point $x$ belongs to the eroded set $X \ominus B^\phi$ iff $x$ and $x+h \in X$. Hence,

$$X \ominus B^\phi = X \cap X_{-h}^\phi \ . \tag{1}$$

The covariance $C(h)$ is given by

$$C(h) = Mes\,[X \ominus B^\phi] \tag{2}$$

where $Mes$ is defined as the digital area of binary object $X$. Hence according to (2), the following properties of $C(h)$ can be derived:

$$C(0) = Mes\,[X] \tag{3}$$

$$C(\infty) = 0 \tag{4}$$

$$C(h) = Mes\,[X \ominus B^\phi] = Mes\,[X \ominus B^{\phi \pm \pi}] = C(-h) \tag{5}$$

$$C(h) \le C(0) \ . \tag{6}$$

### 2.1 Definitions

For a 2-D closed subset $X$ of the Euclidean space $\mathbf{E} = \mathbf{R}^2$, the GCF is defined as

$$K_\phi(h) \triangleq \frac{Mes[X \ominus B_h^\phi]}{Mes[Y]} \tag{7}$$

where $B_h^\phi$ is a two-point structuring element, and $Y$ is a pre-defined standard shape (e.g. a square of size 200×200 pixels), or $Y$ can be chosen to be equal to $X$. For example, the GCF can be restricted to two particular directions, either vertical or horizontal. For this situation, the vertical GCF is defined as follows:



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$$K_{\pi/2}(h) = \frac{Mes\,[X \ominus k_1(h)]}{Mes\,[Y]} \tag{8}$$

where

$$k_1(h) = (\,1 * \cdots * 1)^T \quad ; \tag{9}$$

and the corresponding horizontal GCF is defined as

$$K_0(h) = \frac{Mes\,[X \ominus k_2(h)]}{Mes\,[Y]} \tag{10}$$

where

$$k_2(h) = (\,1 * \cdots * 1) \quad , \tag{11}$$

i.e.,

$$\begin{aligned} k_1(0) &= (\,1)^T & ; \quad k_2(0) &= (\,1) \\ k_1(1) &= (\,1\;1)^T & ; \quad k_2(1) &= (\,1\;1) \\ k_1(2) &= (\,1 * 1)^T & ; \quad k_2(2) &= (\,1 * 1) \\ k_1(3) &= (\,1 * * 1)^T & ; \quad k_2(3) &= (\,1 * * 1) \quad . \end{aligned}$$

### 2.2 Properties

Next, we will investigate some of the basic properties of the GCF as a shape descriptor. These include possible invariance under translation, rotation and scale change. As for any other shape descriptor, these characteristics are important attributes of which one should be aware when using GCF for shape description.

*Property 1*: The GCF is an even positive-valued function.

This is quite obvious since the GCF evaluated at direction $\phi$ and at direction $\phi \pm \pi$ are the same.

*Property 2*: The maximum value of the GCF occurs at the origin, i.e., $K(0) \geq K(h)$.

This is the direct consequence of property 1.

*Property 3*: The area of the shape can be derived from the GCF directly as

$$Are[X] = Mes[X] = K_\phi(0) \times Mes[Y] \tag{12}$$

*Property 4*: The perimeter of the shape can be derived from its GCFs as [7]

$$Per[X] = -\frac{Mes\,(X)}{2} \int_0^{2\pi} K'_\phi(0)\,d\phi \tag{13}$$

*Property 5*: The GCF is invariant under signal translation.

Thus follows since computing the GCF is a morphological operation.

*Property 6*: In general, the GCF is not invariant under signal scale change. However with some normalization, the GCF can be made scale-invariant.

*Property 7*: The GCF alone is not invariant under signal rotation. However, the GCF family is invariant under signal rotation.

*Property 8*: The GCF of a signal is not in general unique to that signal.

This is because the GCF contains only second-order information of a 2-D signal, and hence, the original signal cannot in general be recovered solely from the GCF.

### 2.3 Shape Representation using GCFs

According to the definition of the GCF as indicated by (7), for any deterministic compact set, one can compute a corresponding family of GCFs. Each GCF of this family will represent an angle $\phi$ where $0° \leq \phi < 180°$. Hence, in a discrete domain, the resolution of the GCF representation depends on the step size for the angle $\phi$. The smaller the step size, the higher the resolution of the GCF representation. Of course, this comes at the expense of higher implementation cost.

An example of an object and its GCFs is depicted in Figure 1. Figure 1(a) shows a binary object $X$ which consists of the union of three squares with length $a$ spaced at distance $b$. In Figure 1(b), the GCFs at $\phi = 0$ and $\phi = 90°$ are displayed. The GCF ($K_\phi(h)$) in this case is multiplied by the factor $Mes[Y]$ so that the $K_\phi(0)$ corresponds to the area of the object $X$, i.e., $Are[X]$ ($Mes[X]$) as indicated in the graph. Also, the GCF exhibits successive peaks indicating the periodic nature of the object $X$ in the direction 0°. In fact, when the object is non-convex or is composed of several parts, the correlation at small intervals reveals more information about the individual components than the correlation at large intervals.

Other interesting characteristics of the GCF can be observed from Figure 1. One of these is the *envelope* of the local maxima of the GCF. This *envelope* is shown to be a scaled version of the GCF corresponding to the convex hull of the object in the direction $\phi$ [7]. In addition, the *range* of the GCF in the direction $\phi$ is defined as the smallest $a$ such that $K_\phi(h) = 0$. Thus the *range* can provide an indication of the local geometry of the object.

## 3. SHAPE-RECOGNITION BASED ON THE GCF

In this section, a practical shape-recognition system based on the GCF is presented and results from derived experiments are examined. A simplified block diagram of the proposed shape-recognition system is shown in Figure 2. The major task of this system is to analyze sensed information and to take appropriate action depending on the outcome of the classifier. In this work, we assume that the sensed information is an image which corresponds to the 2-D projection of a real 3-D object.

According to Figure 2, the first step corresponds to the acquisition of the input signal. After acquisition, the incoming signal is digitized through an A/D converter and then applied to the preprocessor. The preprocessor will then provide facilities for enhancement and restoration of the digitized signal. The next step is the major task of feature extraction. This is accomplished by the *morphological correlator* in which the GCFs are computed. However, if we want the system to be size invariant, the object or shape to be described must be pre-scaled before the GCFs are computed. To accomplish this, the 2-D signal that represents the unknown object is linearly and isotropically scaled along the two mutually perpendicular coordinate axes, so that the resulting measure is equal to a pre-defined standard measure.

The GCFs that we obtain from the *morphological correlator* can provide us with information regarding geometrical properties of the unknown object. For example, as noted, both the area (equation (12)) and perimeter (equation (13)) can be deduced from the GCF. In addition, the orientation of the unknown shape can be estimated from the angle associated with the individual GCF. This can be done by comparing the angles associated with the corresponding reference object's GCF and those of the rotated and translated object's GCF. For example, a square rotated by 45° will result in a GCF (evaluated at 0°) that looks like the GCF for the reference square evaluated at 45°. Hence, the difference between the angles associated with the two GCFs of the same shape gives an indication of the orientation of the unknown object.

According to the block diagram of Figure 2, the next step is to determine whether the system is in a training mode. If this is the case, the appropriate attributes of the reference object are stored in the data base. If not, the output of the *morphological correlator* is passed on to the classifier. The classifier then decides which object is present based on the attributes of the unknown relative to those of the reference set. During the classification step, the descriptor values of the unknown object are compared and matched with the reference values, with classification made according to some decision rules. Finally, the outcome of the classifier is passed on to the last module which performs an action such as sending a feedback signal to a robot or activating some other device.

The *morphological correlator* is the heart of the proposed object-identification system. The correlator basically consists of $m$ parallel computation units and a feature-selection unit. The $m$ parallel units compute the GCF of the unknown 2-D image signal for $m$ different directions. The feature-selection unit selects a small subset, $m_f$ where $m_f \ll m$, among the $m$ functions based on some pre-defined criterion.

One feature subset is based on the maximum and/or minimum area under a member of the family of GCFs of a given object. The area under the GCF provides an indication of the geometrical similarity of an object, i.e., the smaller the area, the bigger the change in correlation as the object is moved. This in turn implies that the dimension of the object in one direction is relatively smaller. Other criteria, such as the slope at the origin of the GCF and/or the number of slope changes, can also be used. While the area under the GCF provides a global view of the geometry of the object, the number of slope changes gives an indication of its local geometry. For example, successive peaks imply that some part of the object is periodic.

## 4 EXPERIMENTAL RESULTS

In this section, the performance of the GCF as a shape descriptor is examined. A criterion based on the area under a single GCF curve is used. The number of computation units, $m = 8$ and the number of feature-functions, $m_f = 1$. Experiments were conducted using the eight different shapes shown in Figures 3 and 4. These shapes were chosen to cover a variety of practical objects that will be encountered in an automated manufacturing environment.

Each of the test objects of Figures 3 and 4 were randomly rotated and translated before they were input to the correlator. The amount of rotation (r) and translation (t) for each object is as follows: *disc* (r: 170°; t: 20, 80); *annulus* (r: 80°; t: -100,0); *square* (r: 220°; t: -90 ,-90); *nut* (r: 45°; t: -56, 89); *ellipse* (r: 40°; t: 50, 50); *triangle* (r: 125°; t: -30, 40); *tee* (r: 100°; t: 75, -60); *E* (r: 50°; t: -75, 0). Comparative experimental results are tabulated in the confusion matrix provided in Tables 1 and 2. The classifier used is a simple deterministic minimum-distance classifier which in this case computes the sum of the squared distances, $d(u,r)$ between the reference GCF, $K_\phi^r(h)$ and the unknown GCF, $K_\phi^u(h)$, i.e.

$$d(u,r) = \sum_{h=0}^{N-1} w(h)[K_\phi^u(h) - K_\phi^r(h)]^2 \quad (14)$$

where $w(h)$ is a weighting factor for the $h$ component of the GCF, and $N$ is the maximum extent of the GCF. For these results, $w(h)$ is set to 1. Tables 1 and 2 are organized in such a way that comparisons (the distance $d(u,r)$) between an unknown object and all the reference objects are listed in a column format. When all these columns are combined , a confusion matrix is formed. Tables 1 and 2 list the results of detection using the maximum feature function and minimum feature function of the GCFs respectively. The criterion for selecting the respective feature functions is based on the area under the GCF curve, i.e., the quantity $A[K_\phi]$ where

$$A[K_\phi] = \sum_{h=0}^{N-1} K_\phi(h) \quad . \quad (15)$$

Hence, the maximum feature function, $K_{\phi_{max}}$ is defined as

$$K_{\phi_{max}} = \{K_\phi : A[K_\phi] \text{ is maximun } \}, \ 0° \le \phi \le 180° \quad (16)$$

Similarly, the minimum feature function, $K_{\phi_{min}}$ is defined as

$$K_{\phi_{min}} = \{K_\phi : A[K_\phi] \text{ is minimum } \}, \ 0° \le \phi \le 180° \quad (17)$$

From Table 1, it is observed that the performance of using the maximum feature function is not acceptable as two objects are misclassified, i.e., object *triangle* is mistaken as object *ellipse*, and object $E$ is mistaken as object *tee*. Table 2 shows that by using the minimum feature function, all randomly rotated and translated unknown objects are identified correctly. One conjecture that can be made from the experimental results is that the minimum feature function is less sensitive to rotation than the maximum feature function and thus, it is inherently more stable and reliable than other members of the same GCF family. However, if the resolution of the angle $\phi$ is increased ( that is if more than 8 directions are used), the performance using the maximum feature function can be expected to improve.

## 5. SUMMARY

In this paper, a new 2-D shape descriptor is introduced and its performance is evaluated using different test objects. This shape descriptor which we have called the *geometrical correlation function* (GCF) is based on the idea of morphological covariance and is related to 2nd order properties of the object. Experimental results show that the GCF has great potential in shape representation of 2-D binary objects. However, extension to gray-scale objects is also possible by using a slightly different definition of the GCF.

The potential application of morphological covariance or GCF is not limited to object identification. Since the GCF contains information about the spatial distribution of features of the object, it is possible to extend the GCF idea to the problem of motion parameters estimation. Theoretical study is currently being carried out in this area.

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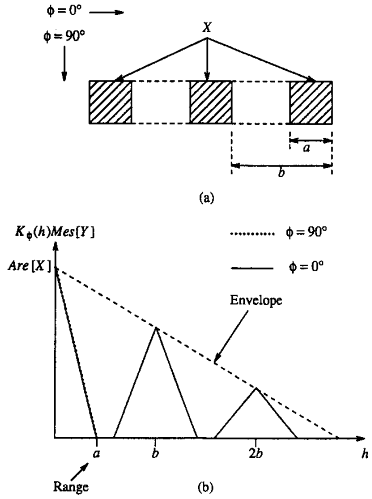


Figure 1 (a) A binary object $X$. (b) Two of its GCFs.

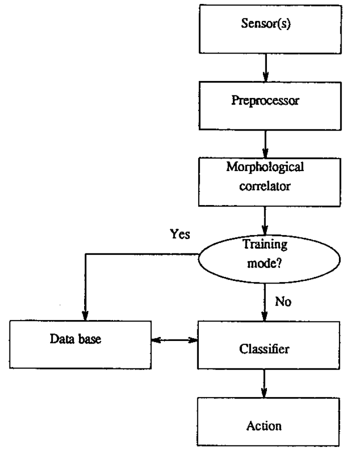


Figure 2 Simplified block diagram of the shape-recognition system.

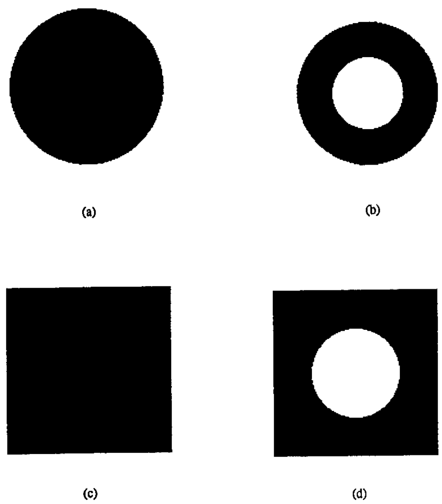


Figure 3 Test objects (area in $pixel^2$) : (a) disk (15623) , (b) annulus (9772), (c) square (22801), and (d) nut (17632).

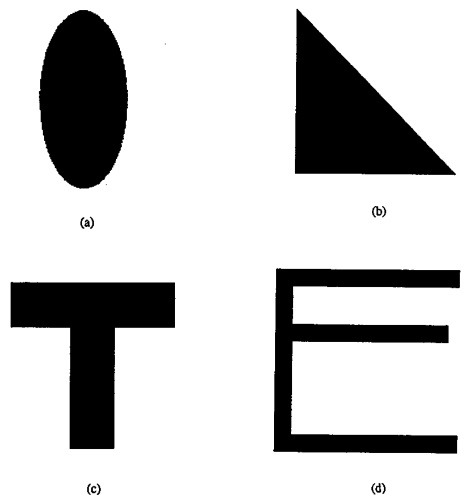


Figure 4 Test objects (area in $pixel^2$): (a) ellipse (10178), (b) triangle (11172), (c) tee (10701), and (d) E (9935).

| Euclidean distances (x10) from translated and rotated shapes to reference shape set | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Shape | disc | annulus | square | nut | ellipse | tri | tee | E |
| disc | 0.00 | 19.34 | 25.42 | 4.16 | 6.10 | 5.59 | 23.47 | 13.66 |
| annulus | 19.38 | 0.00 | 86.15 | 24.97 | 4.80 | 4.64 | 0.43 | 0.97 |
| square | 29.10 | 92.45 | 0.15 | 24.05 | 57.55 | 57.19 | 76.16 | 78.81 |
| nut | 4.13 | 24.88 | 21.19 | 0.00 | 10.45 | 10.06 | 16.81 | 18.77 |
| ellipse | 5.91 | 5.09 | 51.75 | 10.06 | 0.01 | 0.14 | 2.01 | 2.00 |
| tri | 4.73 | 6.85 | 46.51 | 7.58 | 0.37 | 0.40 | 2.99 | 3.16 |
| tee | 10.70 | 1.45 | 65.98 | 15.06 | 1.14 | 0.98 | 0.09 | 0.35 |
| E | 10.55 | 3.45 | 61.42 | 13.22 | 0.84 | 1.16 | 1.38 | 1.03 |
| Decision Correct? | yes | yes | yes | yes | yes | no | yes | no |

Table 1 Confusion matrix for 8 different shapes using only the maximum feature function.

| Euclidean distances (x10) from translated and rotated shapes to reference shape set | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Shape | disc | annulus | square | nut | ellipse | tri | tee | E |
| disc | 0.00 | 19.24 | 20.18 | 5.94 | 19.14 | 15.46 | 17.11 | 35.79 |
| annulus | 19.26 | 0.00 | 75.90 | 13.43 | 1.36 | 1.38 | 0.56 | 2.98 |
| square | 19.78 | 74.95 | 0.01 | 32.88 | 76.05 | 68.65 | 71.84 | 103.65 |
| nut | 6.13 | 12.54 | 34.94 | 0.02 | 11.78 | 9.52 | 10.62 | 24.11 |
| ellipse | 18.85 | 1.39 | 76.43 | 12.36 | 0.00 | 0.20 | 0.27 | 5.19 |
| tri | 16.20 | 1.30 | 71.07 | 10.65 | 0.15 | 0.01 | 0.16 | 5.83 |
| tee | 16.46 | 0.71 | 71.63 | 11.28 | 0.31 | 0.14 | 0.04 | 4.86 |
| E | 36.78 | 3.43 | 106.32 | 26.37 | 5.43 | 6.63 | 4.97 | 0.13 |
| Decision Correct? | yes | yes | yes | yes | yes | yes | yes | yes |

Table 2 Confusion matrix for 8 different shapes using only the minimum feature function.